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The dependence of the dynamo alpha on vorticity

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ABSTRACT

We use data from numerical simulations of dynamo-generated turbulence in the shearing box approximation to determine the dynamo α-effect and its dependence on the rotation law $\Omega(r)$. The data suggest that the dynamo α is not simply proportional to the local angular velocity $\Omega(r)$, as is usually assumed, but rather is proportional to the local vorticity $\omega(r)=r^{-1}\mathrm{d}/\mathrm{d}r(\Omega r^2)$. We also find tentative evidence to support the proposition that the backreaction of the magnetic field on α sets in when the field reaches equipartition with the energy in the turbulent motions. Furthermore, we propose an explanation as to why the sign of α is found to be opposite to that in the standard picture.

Key words: accretion, accretion discs – hydrodynamics – magnetic fields – MHD – turbulence – galaxies: magnetic fields.

1 MOTIVATION

The generation of large-scale magnetic fields in cosmical bodies is often studied using the mean-field approach, where the correlation of small-scale velocity and magnetic field fluctuations, $\boldsymbol{\mathcal{E}}=\langle \boldsymbol{u}'\times\boldsymbol{B}'\rangle$, is approximated in the form

$$\boldsymbol{\mathcal{E}}=\alpha\langle\boldsymbol{B}\rangle-\eta_{\rm t}\nabla\times\langle\boldsymbol{B}\rangle. \qquad (1)$$

For slow rotation, α is proportional to Ω (Steenbeck, Krause & Rädler 1966; Moffatt 1978; Krause & Rädler 1980). A widely used approximation is

$$\alpha\approx-\ell^2\boldsymbol{\Omega}\cdot\nabla\ln\rho \qquad (2)$$

(cf. Krause 1967; Rüdiger & Kitchatinov 1993), where ℓ is the correlation length and ρ the fluid density. For galaxies and accretion discs this may be approximated by

$$\alpha\approx\ell^2\Omega\frac{2z}{H^2} \qquad (3)$$

(e.g. Ruzmaikin, Shukurov & Sokoloff 1988), where H is the Gaussian density scaleheight.

In the classical derivation of α (see e.g. Roberts & Soward 1975) it is usually assumed that Ω is constant, which is often a bad assumption, in particular in systems with strong differential rotation, such as accretion discs and disc galaxies. A simple thought experiment shows that in differentially rotating systems α should depend primarily on the local rate of rotation of fluid elements, and thus on the local vorticity, $\omega(r)=r^{-1}\mathrm{d}/\mathrm{d}r(\Omega r^2)$, rather than just the angular velocity, $\Omega(r)$. Here, r is the distance from the rotation axis in cylindrical polar coordinates, (r,ϕ,z), appropriate for galaxies and accretion discs.

Consider a picture in the spirit of Parker (1955) – see also Moffatt (1978) – Fig. 1. A small fluid element rising a little distance, because of buoyancy for example, will enter regions of lower density and will so expand. Because the angular momentum of the fluid element is conserved, it will begin to spin more slowly about its own axis. Suppose that the bubble is threaded by a horizontal magnetic field, then the field lines will be distorted in the way depicted in Fig. 1. This swirl in the magnetic field lines can lead to a non-vanishing α, i.e. a component of $\langle\boldsymbol{u}'\times\boldsymbol{B}'\rangle$ in the direction of $\langle\boldsymbol{B}\rangle$.

Clearly, α must vanish when there is no rotation at all, i.e. when $\Omega=\omega=0$. Furthermore, when $\Omega\sim r^{-2}$, the fluid has uniform angular momentum and so a patch of fluid rotating about the disc centre has zero rotation with respect to a non-rotating frame. A rising fluid element will expand, but, because its angular momentum is conserved, it will stay non-rotating. There will then be no swirl to magnetic field lines threading the bubble and again no α-effect. More generally, any fluid element will rotate with the angular velocity $\omega/2$ with respect to an observer at rest. So, one would expect $\alpha\propto\omega(r)$, as was assumed already by Donner & Brandenburg (1990). Our aim in this Letter is to point out that this expectation is borne out by numerical simulations of turbulence.

2 SIMULATIONS AND $\alpha\Omega$-DYNAMOS

Computational fluid dynamics has now reached a state where we can simulate large-scale dynamo action, which enables us to estimate the dynamo α, and especially its dependence on ω. In principle one could use a three-dimensional simulation of turbulence and apply a magnetic field, as was done by Brandenburg et al. (1990) for rotating convection and more recently by Tao, Cattaneo & Vainshtein (1993) for forced helical turbulence. However, first of all, in the simulations of Brandenburg et al. (1990) there is only rigid rotation, so the dependence on ω could not have been studied. Secondly, there is a general problem in that the turbulence tends to oppose such an applied magnetic field and expels it into regions of

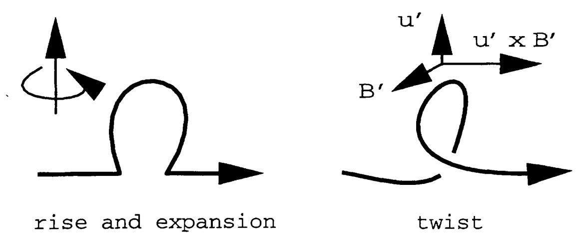

Figure 1. Sketch of a cyclonic event. A fluid element rises and expands, dragging magnetic field lines with it. The field line is then twisted clockwise by the effect of rotation. Note that $\boldsymbol{u}' \times \boldsymbol{B}'$ has a component in the direction of $\langle \boldsymbol{B} \rangle$, so $\alpha > 0$.

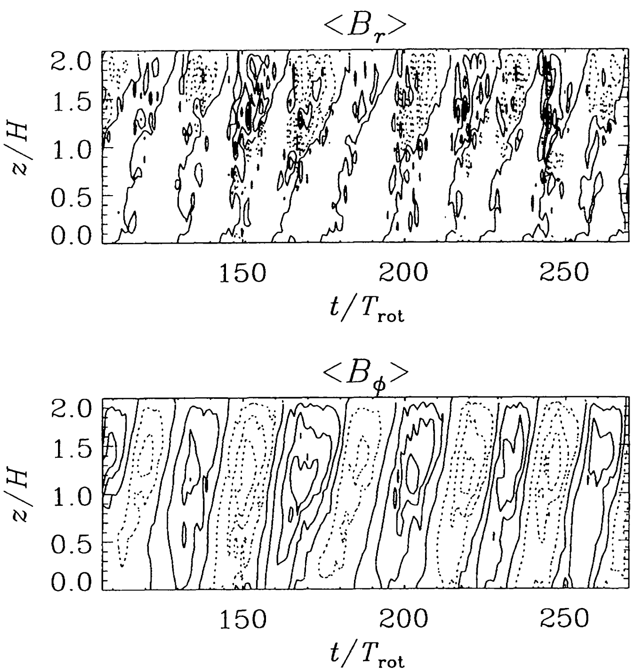

Figure 2. Spatio-temporal pattern of the radial and toroidal components of the large-scale field, $\langle B_r \rangle$ and $\langle B_\phi \rangle$, respectively, as a function of height and time. The data are from Model O of the three-dimensional simulation of Brandenburg et al. (1996).

weak turbulence. In those cases the fluctuations are much stronger than the imposed field. This is in marked contrast to simulations where a large-scale magnetic field is not applied, but is instead generated self-consistently from the turbulent motions themselves. In this case the magnetic energy is equally distributed over all scales. This seems to be typical of dynamos exhibiting large-scale field generation. The non-linear turbulence model of Pouquet, Frisch & Léorat (1976) is such an example, where large-scale fields are generated by an inverse magnetic cascade.

In the following we employ the accretion disc simulations of Brandenburg et al. (1995). Here the rotation is non-uniform, and a large-scale field is generated in the direction of the shear. This large-scale field is comparable in strength to the small-scale fluctuations. In Fig. 2 we show horizontally averaged fields $\langle B_r \rangle$ and $\langle B_\phi \rangle$ as a function of normalized height z/H and time $t/T_{\rm rot}$, where $T_{\rm rot} = 2\pi/\Omega_0$.

The figure shows that there is a well-defined large-scale field (especially in $\langle B_\phi \rangle$) that varies cyclically with a period of about

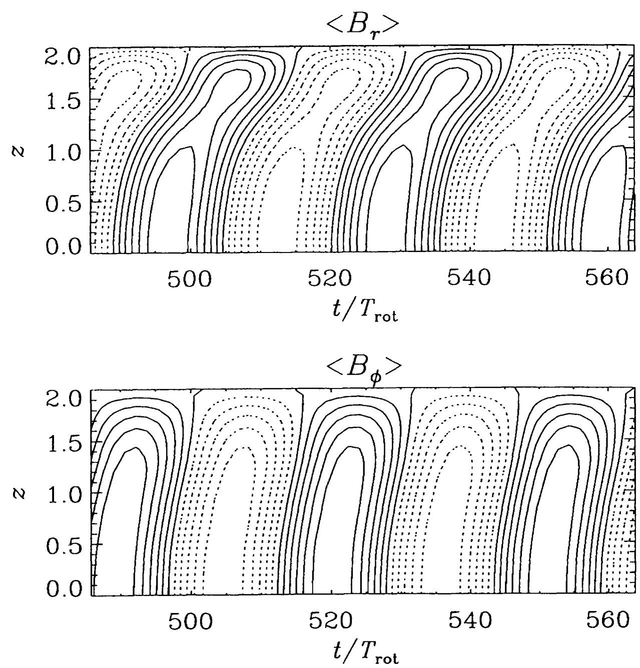

Figure 3. Spatio-temporal pattern of $\langle B_r \rangle$ and $\langle B_\phi \rangle$ as a function of height and time, obtained by solving (4)–(7).

$30T_{\rm rot}$. The field also appears to migrate away from the mid-plane at a speed $V \approx 0.024\Omega H$. We now make the hypothesis that the resulting field can be reproduced by an $\alpha\Omega$-dynamo model in a slab, governed by the horizontally averaged induction equation using equation (1):

$$\frac{\partial \langle B_r \rangle}{\partial t} = -\frac{\partial}{\partial z}\alpha\langle B_\phi \rangle + \eta_{\rm t}\frac{\partial^2 \langle B_r \rangle}{\partial z^2}, \tag{4}$$

$$\frac{\partial \langle B_\phi \rangle}{\partial t} = -q\Omega\langle B_r \rangle + \eta_{\rm t}\frac{\partial^2 \langle B_\phi \rangle}{\partial z^2}, \tag{5}$$

where we have assumed $\eta_{\rm t}$ independent of z, and where $q = 3/2$ for Keplerian rotation. Since $\alpha \ll \Omega H$ we have neglected the α-effect in the second equation (5). On the boundaries we assume

$$\frac{\partial \langle B_r \rangle}{\partial z} = \frac{\partial \langle B_\phi \rangle}{\partial z} = 0 \quad \text{on} \quad z = 0; \tag{6}$$

$$\langle B_r \rangle = \langle B_\phi \rangle = 0 \quad \text{on} \quad z = L_z. \tag{7}$$

This boundary condition was also used in the three-dimensional simulations (except in those cases where no symmetry was prescribed).

The calculations of Brandenburg et al. (1995) confirmed that α changes sign about the equator. The simplest functional form for α is therefore $\alpha = \alpha_0(z/H)$. For $\alpha_0 = -0.001\Omega H$ we reproduce the right cycle frequency, $\Omega_{\rm cyc}/\Omega = 0.03$. In fact, one can show that $\Omega_{\rm cyc}/\Omega = \mathcal{O}(|\alpha/\Omega H|^{1/2})$. The other free parameter is $\eta_{\rm t}$, which we fix by assuming the solution to be marginally excited. This gives $\eta_{\rm t} = 0.0078\Omega H^2$. In Fig. 3 we plot the resulting spatio-temporal pattern of $\langle B_r \rangle$ and $\langle B_\phi \rangle$. The agreement with Fig. 2 is quite striking.

Of course, the $\alpha\Omega$-model is rather simplistic: it ignores the detailed dependences of α and $\eta_{\rm t}$ on z and $\boldsymbol{B}$, as well as anisotropies. The model also does not take account of the fact that in accretion discs the turbulence (which drives α and $\eta_{\rm t}$) is caused by $\boldsymbol{B}$ itself due to magnetic (Parker and Balbus–Hawley) instabilities. We refer to this as dynamo-generated turbulence. However, we feel that the dynamo mechanism here is typical also for other systems where shear is important. In fact, even in the Sun and in galaxies, the α-effect may be predominantly due to magnetic instabilities (Schmitt 1985; Ferriz-Mas, Schmitt & Schüssler 1994; Hanasz & Lesch 1997).

3 DEPENDENCE OF α ON ω AND B

In the following we estimate α directly from $\mathcal{E}$ without invoking the assumption of an underlying $\alpha\Omega$-dynamo. We use data from the three-dimensional calculations, so equations (4)–(7) will not be used. We simply assume $\mathcal{E}_\phi = \alpha\langle B_\phi\rangle$ (to first order) and estimate α from a scatter plot of $\mathcal{E}_\phi \equiv \langle u_z B_r - u_r B_z\rangle$ versus $\langle B_\phi\rangle$ using a least-squares fit (see fig. 8 of Brandenburg et al. 1995). The results are compatible with the estimates obtained from the cycle frequency.

By assuming $\mathcal{E}_\phi = \alpha\langle B_\phi\rangle$ we have ignored other effects such as turbulent diffusion and turbulent fluctuations, which are responsible for the scatter. In a more extensive study Brandenburg & Sokoloff (in preparation) have included the effects of turbulent diffusion, but their results suggest that estimating α from the scatter plot is a reasonable approximation, which will suffice for the purpose of the present paper. Regarding the fluctuations in the relation $\mathcal{E}_\phi$ and $\langle B_\phi\rangle$ we note that even a fluctuating α-effect can lead to large-scale dynamo action owing to the presence of shear (Vishniac & Brandenburg 1997).

In order to study now the dependence of α on the vorticity, we have to vary the rotation law. Abramowicz, Brandenburg & Lasota (1996) have used accretion disc simulations to compute the effective disc viscosity assuming $\Omega \sim r^{-q}$ and varying q between 0.1 and 1.8. We use the same set of data to estimate α for different values of q. The simulations have been carried out in local Cartesian geometry, but for convenience we have translated them into cylindrical polar coordinates, so $B_r \equiv B_x$ and $B_\phi \equiv B_y$, for example. The vorticity is

$$\omega = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\Omega\right) = (2-q)\Omega, \tag{8}$$

and so a range in ω from 0.2Ω to 2Ω can be covered. (For $q=2$ the disc is Rayleigh-unstable and no statistically steady state has been found. For small values of q the shear is too weak and the dynamo is no longer oscillatory and perhaps not even excited. Thus our results for $q \le 0.5$, i.e. $\omega/\Omega \ge 1.5$, should be considered with care.)

The results for all values of q are given in Table 1, together with the average values of square roots of the kinetic and magnetic energies, expressed in terms of $\mathcal{V} \equiv (2E_{\rm kin}/\langle\rho\rangle)^{1/2}$ and $\mathcal{B} \equiv (2E_{\rm mag}/\langle\rho\rangle)^{1/2}$, and the Gaussian density scaleheights H relative to the radial extent L. (H varies because it depends on the temperature which, in turn, results from a balance between viscous and ohmic heating on the one hand and cooling on the other.) The normalized values of α are plotted in Fig. 4. Note that α increases with ω in a way that is consistent with $\alpha \propto \omega$, confirming the ideas outlined above. This is our main result. In the plot the last point (at $\omega/\Omega = 1.9$, i.e. $q = 0.1$), however, clearly deviates from the linear relation. As explained above, this last data point is uncertain. For $\omega \longrightarrow 2\Omega$ we approach the regime of rigid rotation where dynamo-generated turbulence no longer operates. Under different circumstances turbulence could still be driven externally (e.g. by convection), although α may then have the opposite sign, as expected from equations (2) and (3). In other words, α may change sign near $\omega = 2\Omega$.

Table 1. Summary of the normalized values of α for different values of q. Also given are the normalized values of the square root of the kinetic and magnetic energies, and the Gaussian density scaleheight. The values for $q = 0.1$ and perhaps also 0.5 are uncertain (see text).

q	$\alpha/H\Omega$	$\alpha_{r\phi}/H\Omega$	$\mathcal{V}/H\Omega$	$\mathcal{B}/H\Omega$	H/L
0.1	−0.00008	0.0008	0.04	0.12	1.4
0.5	−0.00097	0.0042	0.06	0.15	1.4
1.0	−0.00111	0.0064	0.11	0.23	1.3
1.5	−0.00038	0.0027	0.12	0.21	2.3
1.7	−0.00041	0.0031	0.14	0.20	2.0
1.8	−0.00014	0.0016	0.18	0.26	2.7

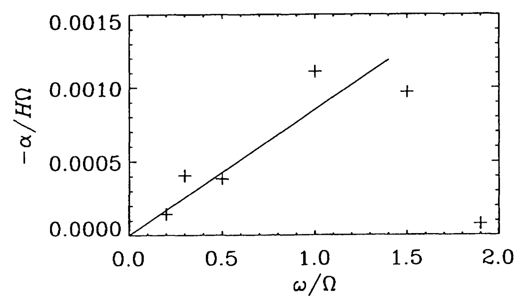

Figure 4. α as a function of ω/Ω. For $\omega/\Omega \le 1.5$ the data points lie roughly on a straight line.

The magnitude of α turns out to be smaller than anticipated in Section 2 based on comparisons with $\alpha\Omega$-dynamos. For $q = 1.5$, for example, this comparison yields the value $\alpha_0 = -0.001\Omega H$, whereas Table 1 gives the value $-0.0004\Omega H$. However, if we correlate $\mathcal{E}_\phi + \eta_{\rm t}(\nabla\times\langle\boldsymbol{B}\rangle)_\phi$ (instead of just $\mathcal{E}_\phi$) against $\langle B_\phi\rangle$ then the resulting slope is closer to -0.001. (Here we have assumed $\eta_{\rm t} = 0.008\Omega H^2$, which is close to the turbulent viscosity of $0.005\Omega H^2$, cf. Brandenburg et al 1996.) Thus the discrepancy can partly be explained by the neglect of turbulent magnetic diffusion. However, since the scatter of the correlation gets worse, we continue to estimate α in the following simply from the correlation between $\mathcal{E}_\phi$ and $\langle B_\phi\rangle$.

In the remainder of this section we briefly discuss further results that are related to the dynamical feedback on α, as well as its anisotropy and sign.

When the field becomes dynamically important, α may no longer be independent of $\boldsymbol{B}$, so the relation between $\langle B_\phi\rangle$ and $\mathcal{E}_\phi$ will no longer be linear. In Fig. 5 such a deviation is clearly seen. Evidently, the deviation sets in when the field becomes comparable to $B_{\rm eq} \equiv \langle 4\pi\rho\boldsymbol{u}^2\rangle^{1/2}$. This is certainly at least in agreement with conventional theories of α-quenching (e.g. Moffatt 1972; Rüdiger & Kitchatinov 1993), and appears to be contrary to the idea that the onset of α-quenching may set in for much weaker fields as the magnetic Reynolds number $R_{\rm m}$ is increased. Vainshtein et al. (1993) have proposed that the onset of α-quenching would already occur near $R_{\rm m}^{-1/2}B_{\rm eq}$. In our cases the average magnetic Reynolds number is about 100, so the onset of α-quenching would be at $0.1B_{\rm eq}$ and not at $B_{\rm eq}$, as in our case. On the other hand, the argument of Vainshtein et al. (1993) does not really apply to the present simulations, because here the magnetic energy is not concentrated at small scales, as explained above. To settle this issue, however, one would really need to calculate α for different values of $R_{\rm m}$. Unfortunately, calculations with higher values of $R_{\rm m}$ are prohibitively expensive in terms of computer time.

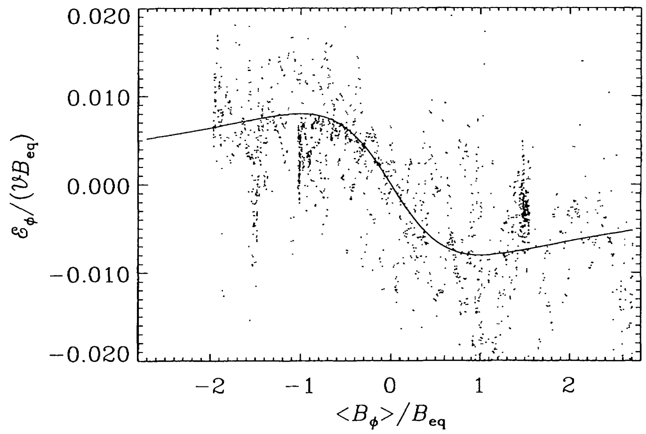

Figure 5. Scatter plot of $\mathcal{E}_\phi$ and $\langle B_\phi\rangle$ at different times for $q = 0.5$. The solid line marks the relation $\mathcal{E}_\phi/\mathcal{V} = -0.016\langle B_\phi\rangle/(1 + \langle B_\phi\rangle^2/B_{eq}^2)$.

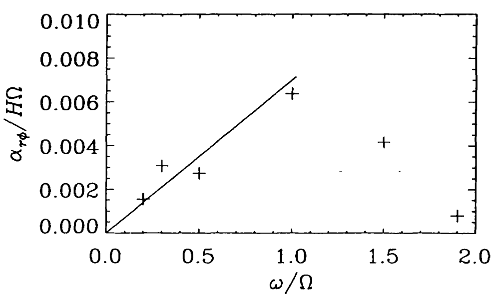

Figure 6. $\alpha_{r\phi}$ as a function of ω/Ω. For $\omega/\Omega \leq 1$ the data points lie roughly on a straight line.

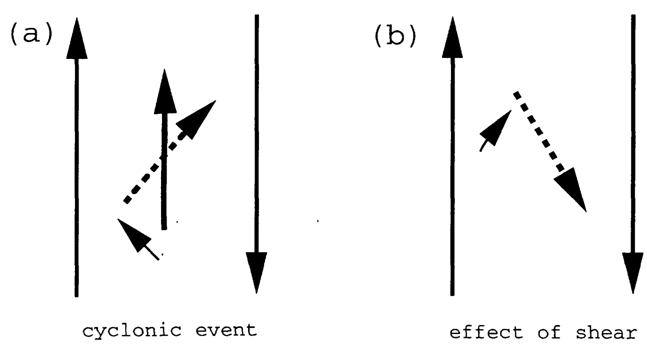

Figure 7. Sketch illustrating the effect of shear on the Parker loop, as viewed from above. ($z > 0$ and $\Omega > 0$.) (a) A cyclonic event twists a toroidal flux tube ($B_\phi > 0$) into the radial direction ($B_r > 0$). Together with buoyancy ($u_z > 0$), this gives $\alpha \sim \mathcal{E}_\phi/B_\phi \sim u_zB_r/B_\phi > 0$. (b) Shear turns the flux tube further by almost 180°, so now $B_\phi < 0$, but B_r and u_z are still positive and thus $\alpha \sim u_zB_r/B_\phi < 0$.

Of course, α is really a tensor. In the analysis above we have focused only on the $\alpha_{\phi\phi}$-component, which is indeed the most important one, as it is responsible for regenerating poloidal magnetic field from toroidal. We can also estimate the $\alpha_{r\phi}$-component from the correlation between $\mathcal{E}_r$ and $\langle B_\phi\rangle$. This component, like several other off-diagonal components, contributes to the antisymmetric part of the α-tensor. It can be written as an effective transport velocity $\gamma = -\alpha_{r\phi}$ in the z-direction (e.g. Rüdiger & Brandenburg 1995). In Fig. 6 we plot $\alpha_{r\phi}$ as a function of ω. For $\omega/\Omega \leq 1$ the two are approximately proportional to each other.

It is difficult to obtain reliable values for the other components of α, because $|\langle B_r\rangle| \ll |\langle B_\phi\rangle|$ and $\langle B_z\rangle = 0$. (We note, however, that α_{rr} seems to be positive and roughly independent of ω/Ω.) However, as mentioned above, those other components are less relevant for the actual dynamo process. We should also point out that Ferrière (1993) has obtained a dependence of the α_{zz}-component on ω, but in her model $\alpha_{\phi\phi}$ was still only dependent on Ω.

Brandenburg et al. (1995) noted that α is negative in the upper disc plane, in sharp contrast to the standard result given in equation (2). The reason lies probably in the importance of shear which turns flux tubes around such that $B_r/B_\phi < 0$ (see Fig. 7). Together with buoyancy ($u_z > 0$), this leads to a dominant contribution to $\alpha \sim \mathcal{E}_\phi/B_\phi \sim u_zB_r/B_\phi < 0$. We note that this effect cannot be explained in terms of a 'Parker loop' inducing a current (cf. Moffatt 1978, fig. 7.2), because this description is static and does not capture dynamical effects related to buoyancy and expansion of the loop. It is not the first time that the static description has broken down. Another example was given by Brandenburg et al. (1990) in the context of convection, where the α-effect in the vertical direction has the opposite sign due to compression of the loop in a downdraught.

In the present paper we have ignored the detailed z-dependence of α, because otherwise we would lose accuracy by no longer averaging over z. However, we still expect an approximately linear dependence on z, so equation (3) should basically be valid, except that Ω should be replaced by $\omega/2$, i.e. $\alpha \propto \ell^2\omega z/H^2$. Furthermore, we now also know that the sign of α is opposite to that in equation (3). Finally, we have seen that the feedback of the magnetic field on the turbulence leads to α-quenching, which is commonly approximated in the form $\alpha \propto 1/(1 + \langle \boldsymbol{B}\rangle^2/B_{eq}^2)$. Thus instead of using equation (3), we propose that for many practical purposes a better formula could be

$$\alpha \approx -\left(\frac{\ell}{H}\right)^2 \frac{\omega z}{1 + \langle \boldsymbol{B}\rangle^2/B_{eq}^2}. \tag{9}$$

Comparing with our simulations, ℓ is of the order of $0.03H$. Note that the effective ℓ is relatively small, so the α-effect seems to be rather weak. However, as discussed above, this is just a manifestation of a relatively long cycle period of 30 orbits. (This is maybe not so unusual: in the Sun the cycle period is 300 orbits!) Also, Brandenburg et al. (1995) estimated that the resulting dynamo number, $q\alpha\Omega H^3/\eta_t^2$, is still large enough for dynamo action; see also Brandenburg & Sokoloff (in preparation).

4 CONCLUSIONS

We conclude that the numerical simulations of dynamo-generated turbulence support the idea that, in situations where the angular velocity Ω varies spatially, α is proportional to the vorticity ω of the background flow, rather than just Ω. This has an effect on the detailed magnetic field distribution in models of galactic dynamos (cf. Donner & Brandenburg 1990). The local rate of field amplification, which depends on the product of α and shear, may be less affected, since in a differentially rotating disc the shear will be reduced where the vorticity is large. In spiral galaxies, Ω is a function not only of r but also of ϕ. In this way ω, and therefore also α, becomes non-axisymmetric. Models of this kind are currently being studied in the case of the bar galaxy M83 (Donner & Brandenburg, in preparation).

Apart from having established an approximately linear dependence between α and ω, we have also found some evidence to support the idea that the α-effect is quenched when the magnetic field approaches the equipartition value. This has been a matter of debate in recent years. The difficulty in clarifying this issue arises from the fact that meaningful estimates of α and its dependence on $\langle \boldsymbol{B} \rangle$ can probably only be obtained in simulations capable of producing a large-scale dynamo. Simulations with perfectly conducting or periodic boundaries are perhaps not suitable, as discussed in the review by Beck et al. (1996). In those situations the mean field is prescribed and cannot change. Furthermore, the mean field is uniform and therefore the diffusion term of the mean field vanishes. Thus, if there were an α in the simulation, it would be forced to have zero effect on the mean field. In our present simulations the mean field is allowed to change and therefore an α-effect, if it is present, can have an effect on the mean field. An important ingredient of a large-scale dynamo is strong shear. The models of Brandenburg et al. (1995) suggest that shear does not only seem to be essential for producing a large-scale dynamo, but also seems to have a profound effect on the *sign* of α.

Our results are well summarized by equation (9). Although this formula was really only obtained in the context of accretion discs, it may well be applicable to galaxies and perhaps even to stars with strong shear layers. [In the latter case $\omega z/H^2$ should be replaced by $-\frac{1}{2}\boldsymbol{\omega}\cdot\nabla \ln \rho$, cf. equations (2) and (3).] However, there are other mechanisms contributing to, or responsible for, the forcing of turbulence: supernova explosions and stellar winds in galaxies, and convection in stars. The effects of those mechanisms on the α-effect remain to be investigated. We cannot claim that our results unambiguously establish the validity of the form of α-quenching included in equation (9), but we do believe that the sort of investigation presented here provides a way to resolve the issue, for example when enhanced computational resources become available.

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