Local models of small-scale dynamo action

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Abstract

Coherent structures in turbulent flows consisting of vorticity filaments may generate small–scale magnetic fields by means of dynamo action with a mechanism similar to that known as the Herzenberg dynamo. In order to analyse the consequences of this assumption for the properties of the resulting magnetic field, we have performed numerical simulations for a Herzenberg–like model system with a prescribed flow, and determined the critical magnetic Reynolds number Hz for the dynamo. The critical value is of the order of a few hundred and is to be compared with the local magnetic Reynolds number for a pair of vortex filaments, calculated as a function of the Reynolds number of the flow by means of a scaling argument following Kolmogorov theory. This gives us fairly stringent conditions on the magnetic Prandtl number. Pr_M has to be large, of the order of a few hundred. The resulting magnetic field has a scale comparable with the diameter of the vorticity tubes and is thus of small–scale, compared to the integral scale of the flow.

1 INTRODUCTION

The existence of coherent structures, such as vorticity filaments, is recognised as being a distinctive feature of fully developed hydrodynamical turbulence. This is seen to be connected to the intermittency properties of the velocity field on scales in the dissipation range [1], [2]. In magnetohydrodynamic flows, such structures are still present (see Figure 1) and the magnetic field is also seen to be concentrated in coherent tube–like structures whose diameter is comparable in size with the dissipative scales [3]. We are interested in addressing the question of whether vorticity filaments might have a role in the generation of small-scale magnetic fields by means of dynamo action, as briefly discussed recently by Brandenburg et al. [4]. We test this hypothesis making use of data from simulations of compressible magnetoconvection [3], in which small-scale dynamo action is observed.

2 THE HERZENBERG-LIKE MODEL

The Herzenberg dynamo consists of two rigidly rotating spheres in a conducting medium. The conductivity is finite and the same inside and outside the spheres. The spheres have a radius a and are separated by a distance $2d$. The magnetic Reynolds number for the Herzenberg dynamo (Herzenberg number) is defined as

$$\mathrm{Hz} \equiv \Omega a^2/\eta, \tag{1}$$

where Ω is the angular velocity of the spheres and η the magnetic diffusivity. The dynamo is excited when Hz exceeds a certain threshold which is a function of the ratio d/a and of the relative orientation

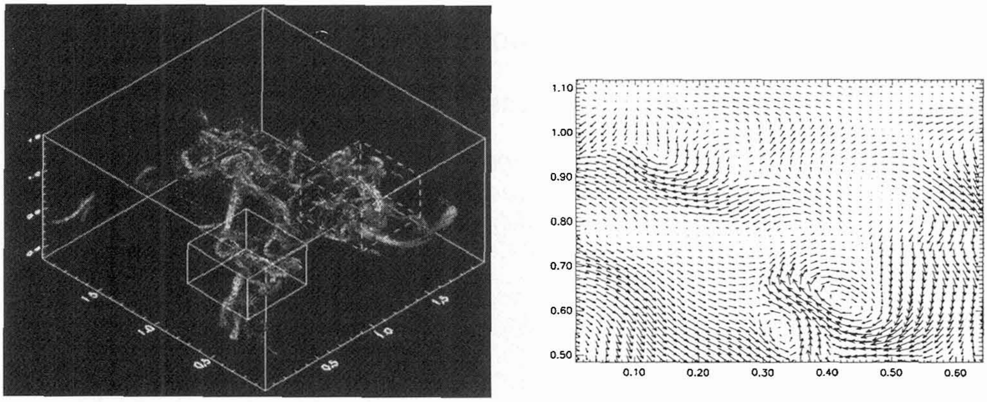

Figure 1: **Left:** Visualisation of the vorticity field in Run D. Vectors whose modulus exceeds three times the rms value are plotted. **Right:** Velocity vectors on a subset of the plane $y = 96$ in RUN D. The two vortex tubes seen at the bottom are considered.

of the two rotation axes. We see from numerical simulations that a similar behaviour is found when the spheres are stretched along their axes of rotation to form ellipsoids. In particular, the critical Reynolds number Hz reaches a limiting value Hz_∞ as the length b of the stretched axis increases. We consider the optimal case when the rotation axes lie in two parallel planes a distance $2d$ apart and inclined to each other by an angle $\phi = 125°$. In the limit $b \gg a$ we take these rotors as a model of vorticity tubes. We find, extrapolating from simulations, that for the case $\phi = 125°$ the condition for the magnetic Reynolds number Hz to be supercritical is

$$\mathrm{Hz} > \mathrm{Hz}_\infty \approx 75 \times (a/d)^{-2.7}. \tag{2}$$

In Figure (2) we show the magnetic field configuration when a dynamo is excited.

2.1 The possibility of small-scale dynamo action

We assume that the length of vortex tubes is comparable to the integral scale L and we make two different assumptions regarding the size of the radius a of the tubes, namely that it is of the order of the dissipation scale $\ell = (\nu^3/\epsilon)^{1/4}$, or that it is of the order of the Taylor microscale $\lambda = \sqrt{5\langle \boldsymbol{u}^2\rangle/\langle \boldsymbol{\omega}^2\rangle}$, where ν is the kinematic viscosity, ϵ is the energy dissipation rate. From the Kolmogorov theory of homogeneous turbulence, these two lengths can be expressed as a function of the Reynolds number of the flow $\mathrm{Re} = UL/\nu$ where U is the rms velocity $\langle \boldsymbol{u}^2\rangle^{1/2}$, so that

$$\ell \approx L\mathrm{Re}^{-3/4} \quad \text{and} \quad \lambda \approx L\mathrm{Re}^{-1/2}. \tag{3}$$

Assuming $a \sim \ell$ and $a \sim \lambda$ in the expression (1) for Hz, we obtain, making use of (3) and dropping factors of order unity,

$$\mathrm{Hz} = \frac{UL}{\eta}\left(\frac{\ell}{L}\right)^{4/3} \approx \mathrm{Pr}_M \ \text{(case I)}. \tag{4}$$

$$\mathrm{Hz} = \frac{UL}{\eta}\left(\frac{\lambda}{L}\right)^{4/3} \approx \mathrm{Re}^{1/3}\mathrm{Pr}_M \ \text{(case II)}. \tag{5}$$

where Pr_M is the Prandtl number $\mathrm{Pr}_M = \nu/\eta$. A comparison of these two estimates of the local magnetic Reynolds number with equation (2) shows that, assuming that on the small scales $a/d \sim O(1)$, there is a possibility of Herzenberg-like dynamo action from vortex tubes only for large magnetic Prandtl number or, from the second relation (5), for very large Reynolds numbers.

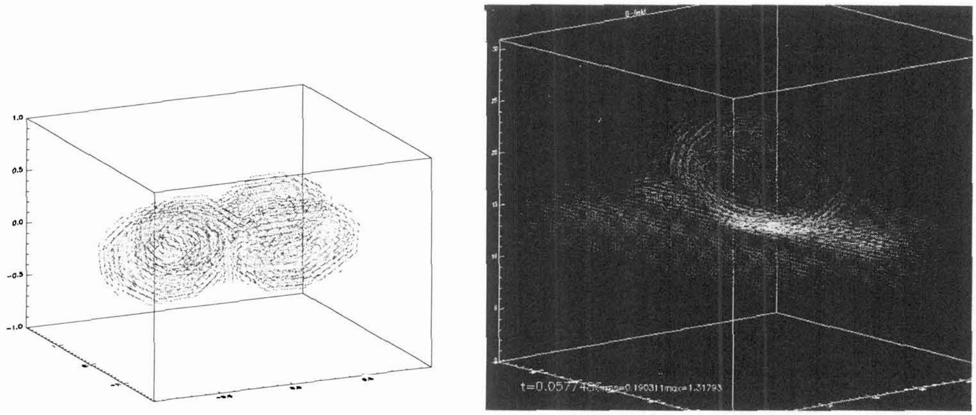

Figure 2: Three dimensional visualization of the magnetic field. Vectors corresponding to fields with strengths exceeding 3 times the rms value are plotted. **Left:** Herzenberg-like dynamo: $\phi = 125°$, $b = 1.8$, $a = 0.3$, $d = 0.4$, Hz = 180. **Right:** The model system of a vortex sheet and one rotor. The rotation axis of the sphere and the vorticity vector of the sheet here lie in the same plane, ($\theta = 90°$) at an angle $\phi = 45°$ with each other.

It is worthwhile noting here that, in the Sun, Pr_M is of the order 10^{-7}, so that this mechanism could not be used as a qualitative picture of the phenomena involved in the small scale dynamo. Moreover, whenever $\mathrm{Pr}_M < \mathrm{Pr}_M^{\mathrm{crit}}$, the mechanism proposed here could not work because then the typical length scale for magnetic diffusion $(\eta^3/\epsilon)^{1/4}$ would be larger than the kinematic dissipation length so that advection of magnetic fields on the scales of the vorticity tubes would be impossible.

2.2 Comparison with turbulent magnetoconvection data

We use now the data from a simulation of turbulent magnetoconvection [3] and in particular data from two different runs which we refer to in the following as Runs A and D. In Run A Re = 310 and $\mathrm{Pr}_M = 4.0$, whilst in Run D Re = 1200 and $\mathrm{Pr}_M = 0.5$ (see table in Figure 3). Looking at flow patterns (Figure 1), we can identify and select a typical vortex pair and measure the Herzenberg number (1); (Figure 3). We see that whilst in Run A Hz is sufficiently large to be supercritical, in Run D it is not.

3 FURTHER MODELS

3.1 Vortex sheets

Following the philosophy of investigating the role of coherent structures in dynamo action, we can look at different structures. An inspection of the flow patterns shows, alongside with vortex tubes, the presence of quasi-2-dimensional structures like downdrafts, where the transverse coherence of the velocity field is well pronounced. We have performed kinematic dynamo simulations for a model system composed of a vortex tube and vortex sheet of the form:

$$\omega_x = -2\frac{u_0}{a^2} z \, \exp[-(z/a)^2], \tag{6}$$

	Run A	Run D
Mesh	63^3	$126^2 \times 105$
L	1	1
Re	310	1200
Pr_M	4.0	0.5
$\langle \boldsymbol{u}^2 \rangle^{1/2} \equiv U$	0.065	0.036
$\langle \omega^2 \rangle^{1/2}$	0.741	0.783
$\ell = L\mathrm{Re}^{-3/4}$	0.013	0.0049
$\lambda = \sqrt{5}U/\langle \omega^2 \rangle^{1/2}$	0.197	0.103

	Run A	Run D
$\Omega = \omega/2$	1	1.4
a	0.07	0.03
d	0.07	0.03
η	5.0×10^{-5}	6.2×10^{-5}
Hz	100	20
$\mathrm{Hz}_{\mathrm{crit}}$	75	75

Figure 3: **Left**: Parameters of Runs A and D **Right**: Vortex tubes. Measured parameters for Run A and D.

so that the velocity takes the form $\boldsymbol{u} = (0, u_y, 0)$, where $u_y = u_0 \exp[-(z/a)^2]$. For such a system we can define a magnetic Reynolds number Sh

$$\mathrm{Sh} = \left[\left(\frac{Ua}{\eta} \right) \left(\frac{\Omega a^2}{\eta} \right) \right]^{1/2}. \tag{7}$$

The growth rate of the magnetic field depends on the relative orientation of the rotation axis of the rotor and the vorticity vector of the sheet and can be positive. In our most favourable case, when the two vectors lie in two parallel planes with relative angle $\phi = 45°$, the critical value is

$$\mathrm{Sh}_{\mathrm{crit}} = 30. \tag{8}$$

The structure of the magnetic field resulting from dynamo action, is shown in Figure 2. Making use of the formula (7) and measuring the parameters in some arbitrarily chosen location in Run D, we have $\mathrm{Sh}_{\mathrm{Run\ D}} \approx 60$, which is of comparable magnitude to the critical value (8).

4 CONCLUSIONS

We have seen that, although tempting visually, it is difficult to explain small-scale dynamo action as a direct result of the interaction between coherent structures in the turbulent flow, especially because of the restrictions on the values of the magnetic Prandtl number. Another difficulty lies in the fact that magnetic field produced exists only on the smallest scales, and thus cannot explain the spectrum of the magnetic fields. In the case of Run D this had a behaviour compatible with a k^{-1} law, meaning that magnetic energy is equally distributed on all scales. Still, it is conceivable that vortex filaments may play a role by producing a magnetic field with r^{-1} dependence, thus giving a k^{-1} spectrum via a mechanism similar to that discussed for the hydrodynamical case by Passot et al. [5].

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