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Intermittent behaviour in axisymmetric mean-field dynamo models in spherical shells

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ABSTRACT

Axisymmetric mean-field dynamo models in spherical shells are shown to be capable of producing temporally intermittent behaviour. This is of potential importance since (i) it is, as far as we are aware, the first time such behaviour has been produced internally by a mean-field dynamo model in a spherical shell, without requiring any additional assumptions or truncations, and (ii) it may be characteristic of the type of behaviour observed in the long-term record of solar activity, such as Maunder minima. We also show that these types of behaviour persist when the functional form of the alpha quenching is altered and also occur over intervals of the shell thickness and the dynamo number.

Key words: chaos – Sun: activity – stars: activity – stars: magnetic fields.

1 INTRODUCTION

There is evidence for solar and stellar variability on a variety of time-scales. In particular, in addition to the (nearly) cyclic magnetic solar cycles (with an average period of about 22 yr) these is also evidence for longer time-scale variations, of the order of 10^2 yr, as typified by the case of the Maunder minimum (Ribes & Nesme-Ribes 1993), during which the amplitude of solar activity, as measured by the sunspot number, was dramatically reduced for about 70 yr. The evidence for this event comes from a number of sources, including the historical data, based on the observations of the sunspot cycles (Eddy 1976), as well as proxy evidence obtained from the record of past ^{14}C variations in the atmosphere, as deduced from the analysis of tree rings (Damon 1976). More recent evidence comes from the ^{10}Be data of ice cores drilled in Greenland (Beer et al. 1990, 1994a,b). The results are remarkable, because they show that even during the Maunder minimum there was still a clear cyclic modulation, indicating that the solar dynamo never shut off completely, but rather that it had become weak enough so that sunspots could not emerge. Furthermore, analysis of sunspot observations made at Meudon Observatory during the time of the Maunder minimum revealed that sunspots erupted only on one hemisphere (Ribes & Nesme-Ribes 1993). This type of behaviour can be interpreted as a mixed parity mode (Nesme-Ribes et al. 1995). A trend for long-term variations of the degree of asymmetry has already been seen in the Greenwich sunspot data (see Brandenburg et al. 1989c).

There is also some direct observational evidence for similar variability in stars (Baliunas & Vaughan 1985; Baliunas et al. 1995). A crucial point here is that ^{14}C data indicate that similar minima have also occurred in the Sun in the past at seemingly irregular intervals (Stuiver & Braziunas 1989).

A great deal of effort has gone into understanding these modes of behaviour in the Sun and stars. At first the main thrust of the modelling was towards explaining the gross properties of the solar cycle, as evidenced by the sunspot cycle: primarily the period and the direction of migration of the dynamo wave ('butterfly diagram'). Notwithstanding the increasing physical content of the models (e.g. Gilman 1983; Brandenburg, Moss & Tuominen 1992), a convincing model that accounts for the equatorward motion of the activity belts during a cycle remains elusive. More recently the generic long-term behaviour of the solar activity has attracted more attention (but see Ruzmaikin, 1981, for an early paper on the subject). The question is how to account for this intermediate, seemingly irregular behaviour in the Sun and stars. There are essentially two approaches to this problem, viewing it either as a deterministic or a stochastic process. In practice, however, the limited length of solar and stellar observations makes this issue rather difficult to decide (Weiss 1990). Here we shall concentrate on the former approach. We comment that, without considerations of non-linearity, variability over these time-scales is difficult to understand in terms of the time-scales

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characteristic of the physical processes thought to be operative in the Sun and stars, at least on the basis of commonly accepted solar and stellar models. The main idea underpinning the deterministic ways of understanding this mode of behaviour is to rely on intuitions obtained from dynamical systems theory and argue that, as solar and stellar dynamos are non-linear systems, behaviour characterized by different statistics over different time intervals may occur and this could account for such variability (e.g. Tavakol 1978; Zeldovich, Ruzmaikin & Sokoloff 1983; Weiss, Cataneo & Jones 1984; Spiegel 1985).

This type of behaviour has subsequently been termed *intermittency*, in analogy with the intermittency in turbulent fluids discovered by Batchelor & Townsend at the end of the 1940s (Batchelor & Townsend 1949). Pomeau & Manneville (1980) described intermittency as a periodic signal interrupted by bursts of irregular behaviour, and this definition has since been generalized to encompass several related behaviours; see for example the discussion in Brooke et al. (1998). Since the 1970s, many models have been constructed showing a variety of types of intermittent behaviour. These range from the classic Pomeau–Manneville intermittency where the dynamics vacillates between nearly periodic and non-periodic regimes, to the crisis intermittency of Grebogi et al. (Grebogi, Ott & Yorke 1983; see also Covas & Tavakol 1997 for a dynamo example), in which the system vacillates between chaotic and periodic or unstable fixed-point regimes, and to the recently described 'on/off' mechanism (Platt, Spiegel & Tresser 1993a; Ott & Sommerer 1994; Ashwin, Buescu & Stewart 1996) in which an attractor in the invariant submanifold loses transverse stability and hence forces the solution to diverge from its 'off' phase (see Covas, Ashwin & Tavakol 1997 for a dynamo example).

A variety of deterministic models have produced behaviour which appears to be (in a qualitative sense) similar to Maunder-type minima in stars with magnetic cycles similar to the Sun. These include:

(1) low dimensional (ordinary differential equation) models, derived either from severe truncations of various approximations to the mean-field dynamo equations (e.g. Weiss et al. 1984; Platt, Spiegel & Tresser 1993b; Covas & Tavakol 1997; Covas, Ashwin & Tavakol 1997); or from normal form theory (e.g. Tobias, Weiss & Kirk 1995); or by considerations of resonances and symmetry breaking (Knobloch & Landsberg 1996);

(2) numerical solutions of the relevant partial differential equations, which take two forms: integration of the full magnetohydrodynamical equations (Gilman 1983), or the integration of mean-field models such as the torus models considered by Brooke & Moss (1994, 1995), solar-type models in Cartesian geometry (Tobias 1997), the models of accretion discs considered by Torkelsson & Brandenburg (1994), as well as models based on amplitude modulations by Tobias (1996, 1997) and Tobias & Weiss (1997).

In this paper, we show that the axisymmetric mean-field dynamo equations, when solved in a spherical shell with an α-quenching non-linearity, without any external driving or perturbation, can also exhibit intermittent-type behaviour. We also that note this behaviour is quite distinct from the chaotic solutions presented, for example, by Tavakol et al. (1995) (hereafter referred to as TTBMT) and that as far as we are aware, this is the first time that such behaviour has been observed in mean-field spherical shell dynamo models and thus these results may be of potential importance in providing a connection between mean-field dynamo models and the dynamical behaviour of the solar cycle. We also note that in Schmitt, Schüssler & Ferriz-Mas (1996) the dynamo is stochastically perturbed and this is therefore a stochastic model. Further, we briefly study the question of likelihood of such modes of behaviour as a function of changes in the thickness of the shell and the dynamo number, and identify intervals where such behaviour is present. We also briefly discuss the relevance of a recently recognized type of intermittency, termed the 'icicle intermittency' by Brooke & Moss (1995), Brooke (1997a,b). This is explored in detail in Brooke et al. (1998).

2 TWO-DIMENSIONAL MEAN-FIELD DYNAMOS

The standard mean-field dynamo equation (Krause & Rädler 1980) is of the form

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B} + \alpha \boldsymbol{B}) - \nabla \times (\eta_{\mathrm{t}} \nabla \times \boldsymbol{B}), \tag{1}$$

where $\boldsymbol{u}$ and $\boldsymbol{B}$ are the mean velocity and magnetic fields. As usual, η_{t} is a turbulent diffusivity, which we take to be a constant. We assume the non-linearity to be of α-quenching type, crudely representing the dynamical feedback of the Lorentz force on the small scale motions. In order to verify that our results do not depend generically on the exact form of this non-linearity (particularly as its functional form is not well established), in addition to the commonly adopted

$$\alpha = \frac{\alpha_0 \cos\theta}{1 + \boldsymbol{B}^2}, \tag{2}$$

we also considered two other functional forms of α that have the property $\alpha \sim |\boldsymbol{B}|^{-3}$ as $|\boldsymbol{B}| \rightarrow \infty$ (Moffatt 1972). The first expression is really just an interpolation formula (see also TTBMT) that satisfies the correct asymptotic behaviour, and was originally derived in the context of Λ-quenching by rapid rotation (Kitchatinov 1987), so we replaced the angular velocity by $|\boldsymbol{B}|$:

$$\alpha = \frac{\alpha_0 \cos\theta}{\boldsymbol{B}^4} \left(\frac{3 + \boldsymbol{B}^2}{1 + \boldsymbol{B}^2} + \frac{\boldsymbol{B}^2 - 3}{|\boldsymbol{B}|} \tan^{-1} |\boldsymbol{B}| \right). \tag{3}$$

The second expression we adopted is a result of Rüdiger & Kitchatinov (1993) and has been derived using first-order smoothing:

$$\alpha = \frac{15}{32} \frac{\alpha_0 \cos\theta}{\boldsymbol{B}^2} \left[1 - \frac{4\boldsymbol{B}^2}{3(1 + \boldsymbol{B}^2)^2} - \frac{1 - \boldsymbol{B}^2}{|\boldsymbol{B}|} \tan^{-1} |\boldsymbol{B}| \right]. \tag{4}$$

In each case α_0 is a constant.

We restricted our investigation to purely axisymmetric solutions of equation (1). The code is described in detail in Brandenburg et al. (1989a,b), and is as implemented by TTBMT. We assume a uniform radial rotational shear, Ω_0', and the two dynamo numbers that control the system are $C_\omega = \Omega_0' R^2/\eta_{\mathrm{t}}$, $C_\alpha = \alpha_0 R/\eta_{\mathrm{t}}$, where R is the radius of the outer boundary. Our unit of time is a global diffusion time, R^2/η_{t}. The other two input parameters are the fractional radius of the inner radius of the shell, r_0, and the quantity F that controls the exact form of the inner boundary condition. More precisely, the outer boundary condition was taken to be that the field fitted smoothly on to a vacuum exterior solution, and the conditions at the lower boundary were taken to be a superposition of perfectly conducting and penetrative magnetic boundary conditions in the forms

$$(1 - F)a + F\left(\frac{\partial a}{\partial r} - \frac{a}{\delta_a} \right) = 0, \tag{5}$$

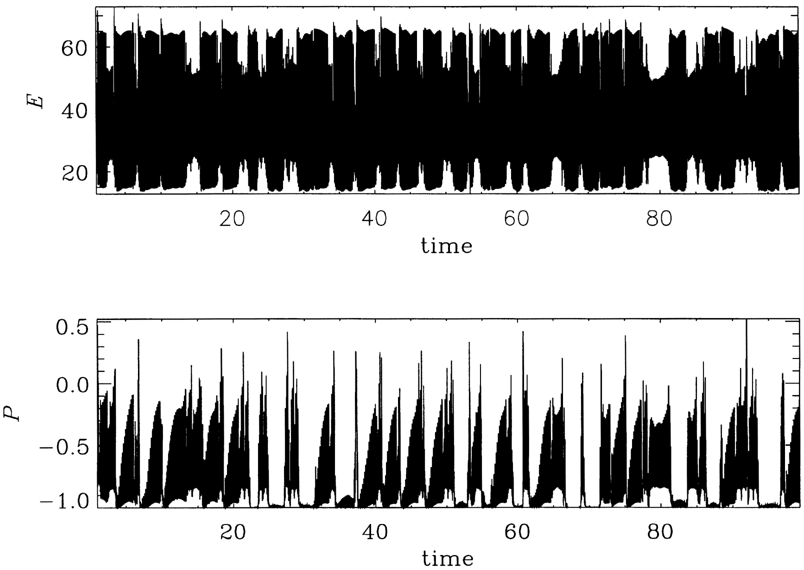

Figure 1. Total energy and parity for the α-profile given by (2). $C_\alpha = 2.0$, $r_0 = 0.2$, $F = 0.59$.

for the poloidal field, and

$$(1-F)\left[\frac{1}{r}\frac{\partial(rb)}{\partial r} - \alpha\frac{1}{r}\frac{\partial(ra)}{\partial r}\right] + F\left(\frac{\partial b}{\partial r} - \frac{b}{\delta_b}\right) = 0, \qquad (6)$$

for the toroidal field. δ_a and δ_b are small distances over which the skin effect can be assumed to reduce the field outside of the convection zone to nearly zero, and we set $\delta_a = \delta_b = 0.05R$. Purely perfectly conducting and penetrative boundary conditions can be recovered by setting F to be 0 and 1 respectively. Our motivation for this unconventional boundary condition is that the true boundary condition is quite uncertain, this being associated with well-known uncertainties about the physical conditions at the boundary of a stellar convective zone. We took $C_\omega = -10^5$ for the calculations described here. A dynamo is excited when $C_\alpha > C_{\alpha c}$. With this value of C_ω, $C_{\alpha c}$ is a somewhat less than 0.1, depending on the exact choice of parameters. The models discussed in this paper are always substantially supercritical, i.e. $C_\alpha \gg C_{\alpha c}$.

As noted earlier, TTBMT found chaotic, but not intermittent solutions. The input parameters of the solutions discussed here differ from those of TTBMT in values of F, C_α and r_0. We have not attempted a survey of the parameter space, and just report interesting behaviour that we encountered.

The behaviour of the models is described by the total magnetic energy in the shell $r_0R \le r \le R$, given by $E = E^{(A)} + E^{(S)}$, and the global parity $P = [E^{(S)} - E^{(A)}]/E$; $E^{(A)}$ and $E^{(S)}$ are respectively the energies of the parts of the magnetic field that are antisymmetric and symmetric with respect to the rotational equator (‘dipole-like’ and ‘quadrupole-like’). Then $P = -1$ denotes an antisymmetric pure parity solution and $P = +1$ a purely symmetric solution.

3 INTERMITTENT-LIKE BEHAVIOUR IN SPHERICAL SHELL MODELS

Within the context of dynamical systems theory, *intermittency* as discussed in the Introduction, comes in a variety of forms, each with precise signatures and underlying dynamical mechanisms. However, in the case of partial differential equations, the characterization of such behaviour is not so well defined. However, attempts have been made to characterize such behaviour in particular cases (Brooke et al. 1998). Here, our aim is mainly to report the existence of such behaviour, rather than its precise identification, and we therefore pragmatically identify intermittency as being characterized by different statistics of the solutions over different time intervals.

In this section we show examples of intermittent behaviour in the dynamo models described in the previous section, with different forms of α-quenching.

Fig. 1 shows an example of a case with the usual form of α-quenching given by equation (2) with parameter values $r_0 = 0.2$, $C_\alpha = 2.0$ and $F = 0.59$. Intermittent behaviour in the parity can be clearly seen, with intervals where P is very nearly -1 (antisymmetric) interspersed with intervals during which the parity migrates past zero and becomes positive. This behaviour is also present in the dynamics of the energies $E^{(A)}$ and $E^{(S)}$, in which the excursions of the parity from $P = -1$ are associated with bursts in $E^{(S)}$.

To study how robust this type of behaviour is with respect to changes in the precise form of the α-quenching, we used other functional forms for α. Thus in Fig. 2 we took the functional form given by equation (3) and parameters $r_0 = 0.4$, $C_\alpha = 1.5$ and $F = 0.7$. Fig. 3, for which equation (2) was also used along with $C_\alpha = 2.04$ and $r_0 = 0.2$, shows that this behaviour can also be occur with a ‘pure’ inner boundary condition such as $F = 1$. As can be seen, these two figures share similarities with Fig. 1, except that now the almost stable state has parity $P = +1$. Finally in Fig. 4 we used the α profile given by (4) with the parameters $r_0 = 0.5$, $C_\alpha = 2.05$ and $F = 0.71$. The behaviour here is rather different in nature from that seen in the previous two figures, in that for most of the time the parity fluctuates over the whole range $-1 < P < +1$, and there are only relatively brief intervals where the energy is nearly periodic and $P \approx +1$.

These figures show that intermittent-type behaviour can occur with different forms of α-quenching, but with different values of input parameters r_0, C_α and different boundary conditions given by the parameter F. They also have different dynamical details. In particular, Figs 1, 2 and 3 display essentially similar behaviour. Fig. 2, however, could be described as an ‘icicle’ intermittency owing to the similarity in form to the parity variations found by Brooke & Moss (1994, 1995). On the other hand, Fig. 4 may turn

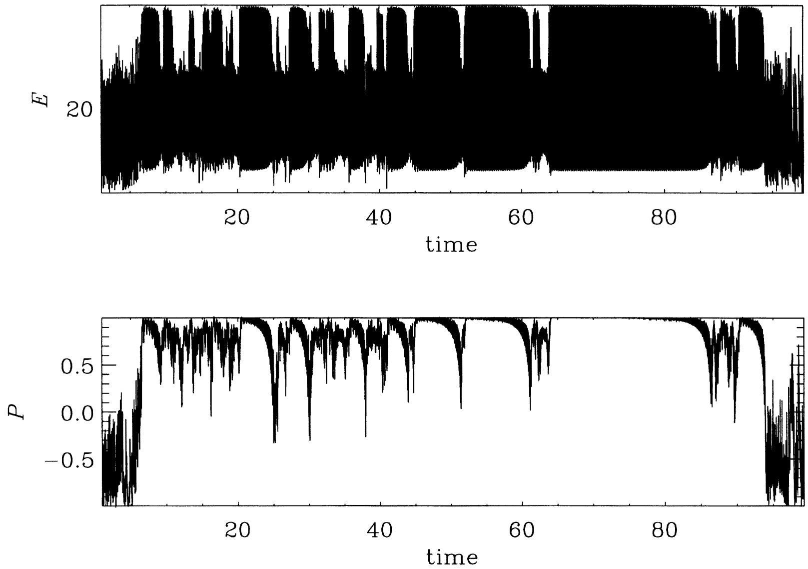

Figure 2. Total energy and parity for the α-profile given by equation (3). $C_\alpha = 1.5$, $r_0 = 0.4$, $F = 0.7$.

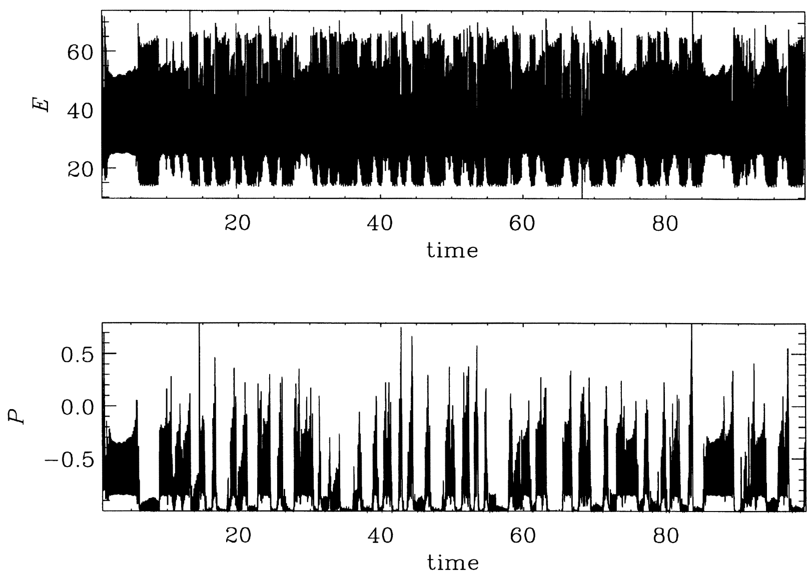

Figure 3. Total energy and parity for the α-profile given by (2). $C_\alpha = 2.04$, $r_0 = 0.2$, $F = 1$.

out to be more representative of an on–off intermittency. In the present context, the invariant submanifold in this example would be defined by quadrupolar ($P = 1$) parity and the transverse perturbations are field components with dipolar parity. This point is worth stressing, because a naive view of on–off intermittency can lead to the conclusion that the magnetic field strength must be much reduced during the 'off' phase. In Fig. 4 the bursts from the $P = 1$ subspace do not have the modulated form shown in the 'icicles' of Figs 1, 2 and 3 and thus are more typical of the on–off mechanism. The point is more fully discussed in Brooke et al. (1998) where it is shown that there are interesting, and perhaps fundamental, links between on–off and icicle intermittency.

Intermittent behaviour was also found for pure inner boundary conditions, with the usual form for α-quenching given by equation (2) being used. Fig. 5 shows such behaviour for an $F = 0$ inner boundary condition with $C_\alpha = 1.95$ and $r_0 = 0.4$.

It is instructive to compare the intermittent-types of behaviour found here with the chaotic and periodic extremes. This progression for a fixed value of $C_\alpha = 2.0$ with α-quenching given by equation (2) and with $F = 1$ can be seen in Figs 6 to 8 in which only the shell thickness r_0 changes. It can be seen that the mean interval between intermittent bursts decreases, until eventually the behaviour becomes chaotic.

Intermittency appears to occur in spherical shell dynamos only when they are very supercritical (ie $C_\alpha \gg C_{\alpha c}$), in contrast to the torus dynamo of Brooke & Moss, where intermittency is found at dynamo numbers only about three times greater than the critical value. Additionally, we have weak evidence that values of r_0 smaller than about 0.5 may be necessary.

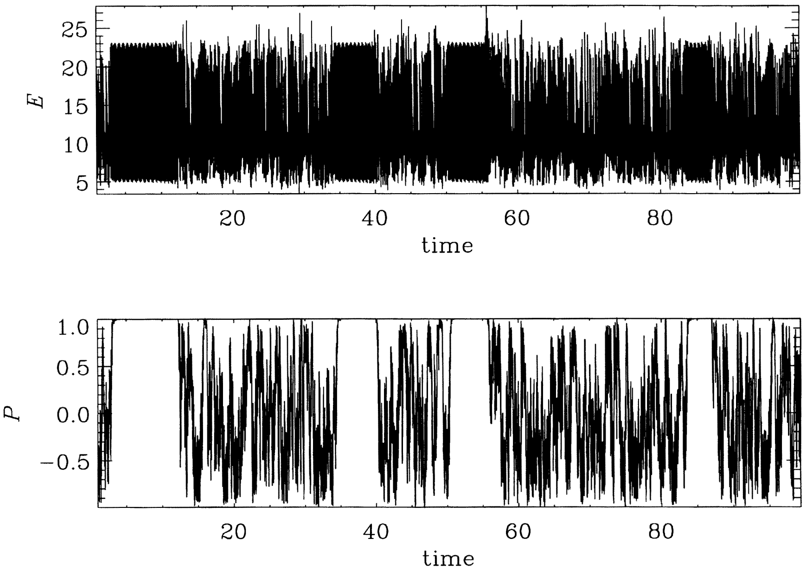

Figure 4. Total energy and parity for the α-profile given by equation (4). $C_\alpha = 2.05$, $r_0 = 0.5$, $F = 0.71$.

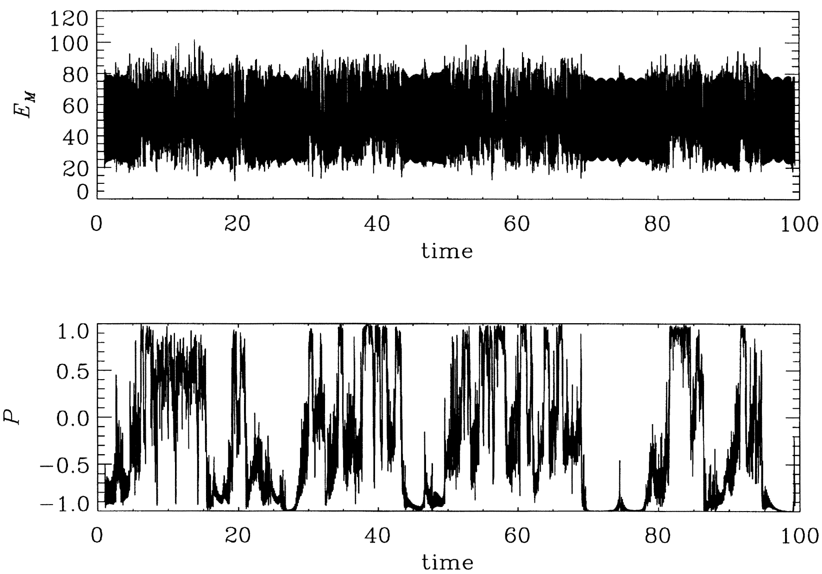

Figure 5. Total energy and parity for the α-profile given by (2). $C_\alpha = 1.95$, $r_0 = 0.4$, $F = 0$.

4 THE LIKELIHOOD OF INTERMITTENT-TYPE BEHAVIOUR

An important question that arises here is how prevalent such intermittent modes of behaviour are, as the various parameters involved are changed. This is of particular importance given that there seems to be observational evidence for intermittent-type behaviour in a variety of stellar settings. It is therefore important that the mechanism that produces intermittent behaviour (whatever its precise dynamical nature may be) should operate in a reasonable range of parameter space.

Here, as a start, we have made a brief study of the likelihood of this type of behaviour as some changes are made in the thickness of the shell r_0 and the dynamo number C_α. We have also considered several values of the boundary condition parameter F.

Table 1 shows the result of our computations regarding variations in C_α versus r_0. Here 'OSM' stands for a mixed parity solution, in which the parity remains positive but displays periodic oscillations and thus a contribution from an antisymmetric (dipolar) solution is present. Similarly, 'OAM' stands for a solution in which the parity is negative, but oscillates periodically. In each case there is usually more than one period present. As can be seen, there are intervals of both r_0 and C_α for which the system shows intermittent-type behaviour. One important outcome of our work seems to be the indication that, at least in these types of mean-field models, intermittent-type behaviour may require thicker shells, with thickness parameter r_0 less than 0.5 or so.

As shown in the previous section, we have also found evidence for generally similar behaviour with 'pure' inner boundary conditions, $F = 0, 1$, as well as intermediate values.

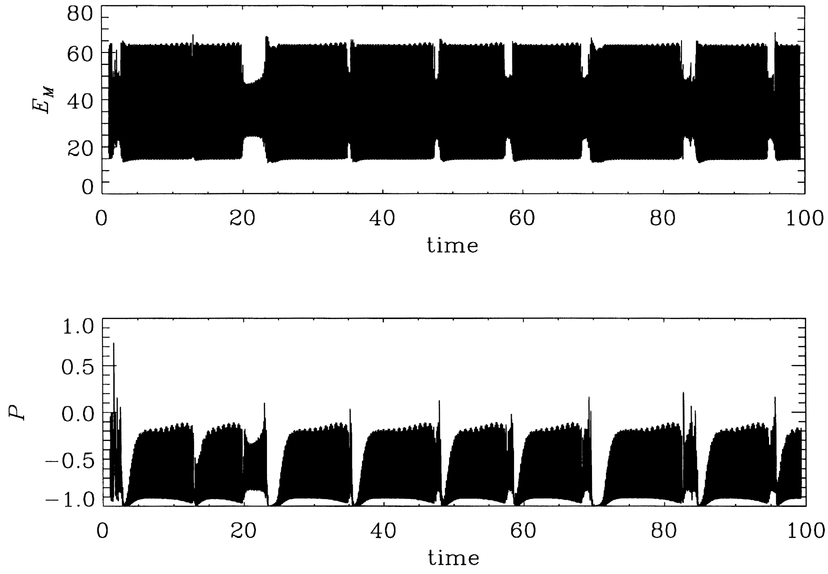

Figure 6. Total energy and parity for the α-profile given by (2). $C_\alpha = 2.0$, $r_0 = 0.18$, $F = 1$.

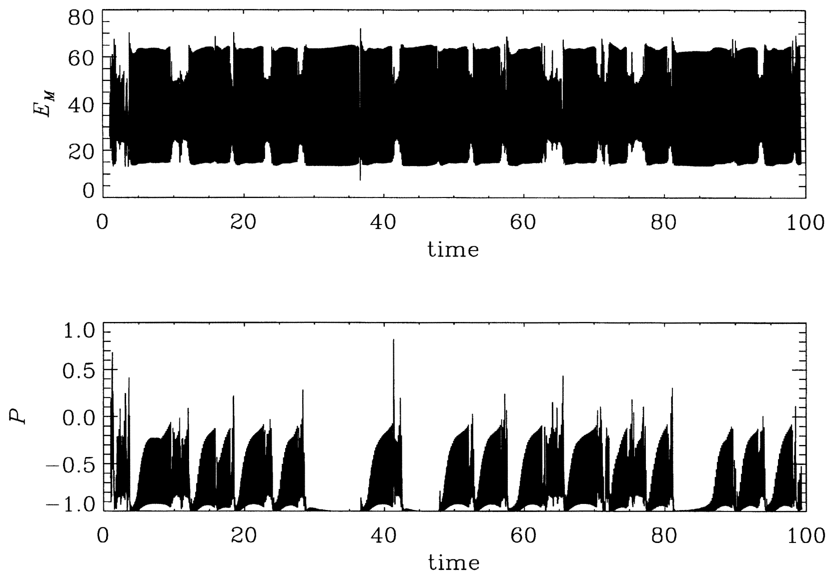

Figure 7. Total energy and parity for the α-profile given by (2). $C_\alpha = 2.0$, $r_0 = 0.19$, $F = 1$.

To gain insight into how the dynamics changes in such models we briefly looked at the way the energy changes as a function of the dynamo number in the model with the α-quenching given by equation (2). The results of our calculations are shown in the top panel of Fig. 9. Initially the stable state of the dynamo changes, at least to the resolution of our parameter grid, from an antisymmetric periodic parity to a symmetric periodic parity solution, without passing through an intermediate mixed parity solution. It then changes abruptly from a symmetric to a mixed antisymmetric periodic parity solution and then to an intermittent phase (we discuss this parameter range in more detail below), after which it becomes a mixed antisymmetric parity solution. It then becomes purely symmetric, $P = +1$, followed by a mixed symmetric parity interval and finally ends up as a chaotic solution. An important point is that the change to the intermittent regime does not involve a dramatic change in the global energy.

Another important feature shown in this figure is the complicated behaviour shown around $C_\alpha \approx 1.35$ and to examine this further, the bottom panel of Fig. 9 shows the behaviour of the models with respect to changing the starting parity in this C_α range. The values used for the starting parity were 0.9999, 0.5, 0.1 (the latter being the value used in all the other calculations present in this paper), −0.5 and −0.9999. As can be seen, there is evidence for multiple attractors by which we mean the existence of solutions having different energies at the same value of C_α but different starting values of the parity, P_i. Our calculations indicate that the pure symmetric solution is unstable to changes in the starting parity, for example by starting with an initial parity value slightly different from 1.0,

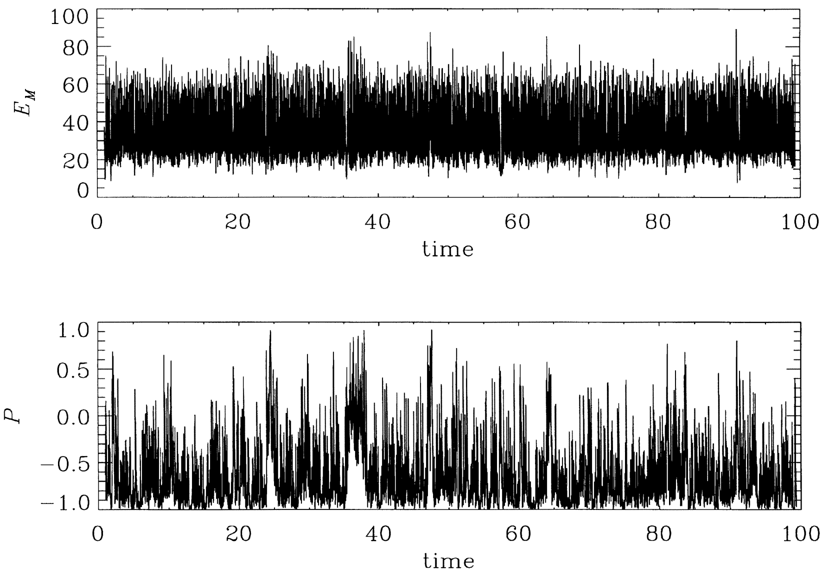

Figure 8. Total energy and parity for the α-profile given by (2). $C_\alpha = 2.0$, $r_0 = 0.25$, $F = 1$.

Table 1. C_α versus r_0 table for α-quenching given by equation (2) with $C_\omega = -10^5$ and $F = 1.0$. Here, S denotes a periodic pure parity symmetric solution; A, a periodic pure antisymmetric solution; OAM, a periodic mixed antisymmetric solution; OSM, a periodic mixed symmetric solution; I, an intermittent and C, a chaotic solution.

C_α r_0	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23	0.24	0.25
1.0	S	S	S	S	S	S	S	S	S	S	S
1.5	A	A	I	I	I	I	C	C	C	C	C
2.0	OAM	OAM	OAM	I	I	I	I	I	I	C	C
2.5	OAM	OAM	OAM	S	OAM	OAM	S	S	S	S	OSM
3.0	OSM	OSM	OAM	OSM	OSM	OSM	S	S	S	OSM	OSM

whilst the intermittent behaviour is apparently more robust with respect to such changes.

5 CONCLUSIONS

We have shown that intermittent-type behaviour can occur in mean-field spherical shell dynamo models with non-linear α-quenching. Our models are deterministic, without the need for ‘outside’ terms to drive the intermittency. The behaviour arises purely from the α-quenching non-linearity. We have shown that such behaviour can be present independently of the detailed form of the non-linear quenching. Further we have found intervals in both the shell thickness and the dynamo number C_α for which intermittency is present. This is important, since to be a physically viable mechanism it is essential that the behaviour survives small changes in the model and the parameters. We also note that this type of behaviour seems to be confined to thick shells with $r_0 < 0.5$.

As far as we are aware this is the first example of the occurrence of intermittent behaviour in such a spherical shell model, without any external assumptions, or truncations to give a low-order system, being made. We note that Brooke & Moss (1995) found some similar behaviour in an axisymmetric model with a torus geometry, and that Tobias (1997) has recently described intermittent behaviour in a related model in cartesian geometry.

We acknowledge that we have not produced a model of the solar cycle. Indeed, that was not our intention but, as mentioned above, we do believe it interesting and important to have demonstrated that mean-field models, of the sort often used to model the solar cycle, can display a variety of intermittent-types of behaviour. *Inter alia*, our models describe distributed dynamos in shells significantly thicker than the solar convection zone and so, if interpreted literally, they would apply to stars of later spectral type than that of the Sun.

We note that we really do not know what signature of the solar cycle we should compare our results with! There are records of the sunspot cycle extending over some 350 years (including by chance the Maunder minimum), which we can assume correlate with properties of the underlying dynamo. However there is no quantitative theory linking the dynamo generated field to the observed spots. As mentioned in the introduction, a dearth of spots almost certainly does not mean that the dynamo generated field is absent throughout the convection zone but, more plausibly, that it is reduced below some threshold value. The same sort of remarks apply to other proxies for the solar cycle.

Of course, another aspect of the astrophysical problem is the behaviour of a stellar dynamo as spin down occurs during the

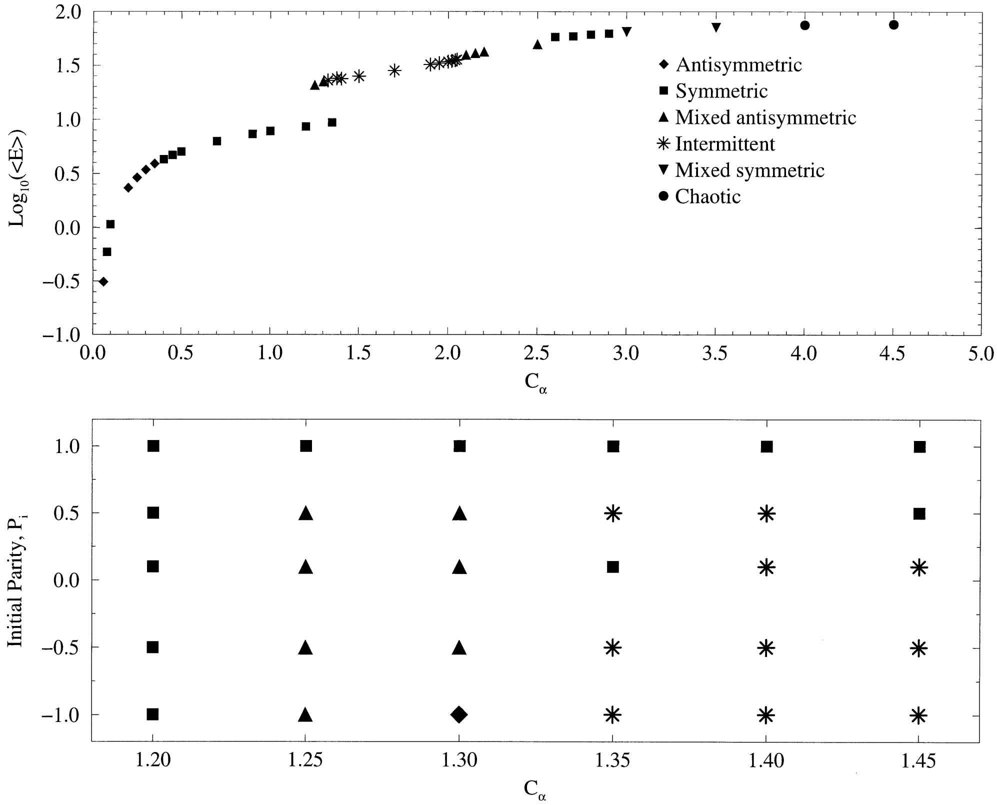

Figure 9. The top panel shows the logarithm of the energy versus C_α diagram for the shell, $r_0 = 0.2$, with α-quenching given by equation(2) with $C_\omega = -10^5$, $F = 1$ and an initial parity of 0.1. The lower panel shows the behaviour of the models depending on the initial parity value for given C_α values. In both figures a diamond indicates a pure antisymmetric periodic parity mode; a filled square, a pure symmetric parity mode; a filled erect triangle, a mixed antisymmetric mode; a star, an intermittent-type like parity mode; a filled inverted triangle, a mixed symmetric parity mode and finally a filled circle, a chaotic parity mode.

evolution of a star. This suggests that it might be interesting to examine changes in $|C_\omega|$. But changes in Ω can be expected to alter the magnitude of the α-effect, and thus C_α, also. Furthermore, r_0 can be expected to change as a star evolves. Thus the precise history of the dynamical state of a stellar dynamo in a star with a deep convective envelope appears hard to determine. We do anticipate, however, that when $|C_\omega|$ has become small enough, the dynamical behaviour is more likely to be regular (i.e. not intermittent or chaotic).

Finally, we mention several recent papers that address specifically the astrophysical aspects of the problem. Tobias (1996) showed that intervals of reduced solar behaviour could be explained as the modulation of a magnetic cycle by the Malkus–Proctor effect (i.e. the feedback of the Lorentz torque on to the differential rotation). This modulation was periodic, but in an extension of this work (Tobias 1997), aperiodic modulation has been found, which thus makes it a possible intermittency mechanism. Schmitt et al. (1996) do present intermittent solutions, but in their work the intermittency arises from the dynamo being stochastically perturbed, modelling the influence of a coupled external system. Knobloch & Landsberg (1996) consider a solar model where resonances and symmetry breaking are key features. A particularly interesting result is that they find two types of grand minima. Type I is where the dynamo spends most of its time in a pure parity state, with occasional excitation of field of the opposite parity, associated with a fall in the total energy. Type II is where the system spends most of the time in a low-energy state, punctuated by exponential rises and sudden falls. During the latter episodes, there are rapid changes in the parity, about the underlying mixed parity state. The behaviour we have described as icicle intermittency appears to be akin to that shown by Type I systems. However, in Knobloch and Landsberg's model the spatial structure of the quadrupolar and dipolar components is fixed and the resonance between them is modelled by two coupled equations for the complex amplitude, which describe the symmetries of the system up to third-order terms. In our solutions the spatial structures of the quadrupolar and dipolar components of the field can evolve, as well as there being evolution due to resonant interactions between them. Resonant modulation is not inevitable, as can be seen from our Fig. 4 where modulation is replaced with sudden and irregular bursts, thus it requires explanation. This issue is explored further in Brooke et al. (1998). We reiterate that the source of the intermittency in our solutions appears to be distinct from anything discussed in these previous papers.

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