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# Comment on "The sunspot as a self-excited dynamo"

**David Moss**[1] **and Axel Brandenburg**[2]

[1] Mathematics Department, University of Manchester, Manchester M13 9PL, UK
[2] Mathematics Department, University of Newcastle, Newcastle upon Tyne NE1 7RU, UK

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**Abstract.** A recent paper claims that the well known Cowling 'anti-dynamo' theorem is a "misconception", and that a simple axisymmetric sunspot model constitutes a counter example. We do not believe these claims to have been substantiated.

**Key words:** Magnetohydrodynamics (MHD) – Sun: magnetic fields – Sun: sunspots

## 1. Introduction

The MHD literature contains a number of 'anti-dynamo' theorems, which have proved important in astrophysical and geophysical applications. The first of these theorems was established by Cowling (1934), and can be simply stated as 'A stationary axisymmetric MHD dynamo cannot be driven by strictly axisymmetric motions' (see also Ch. 18 of Parker 1979). A recent paper by Lorrain & Koutchmy (1998; hereafter LK) contains the sentences "This dynamo is axisymmetric which is forbidden by the Cowling 'theorem' (Cowling 1934, 1957, 1976). However it has long been clear that the 'theorem' is a misconception." The authors proceed to construct a simple, completely axisymmetric, model for a sunspot, which they claim to be a steady self-excited MHD dynamo.

These statements are so remarkable, and potentially misleading, as to require some response. In support of their claims quoted above, LK cite six references. We examine each in turn.

- Kolm & Mawardi (1961) study MHD flows in the presence of a solenoid – there is no claim of dynamo action.
- Shercliff (1965) actually presents an outline proof of Cowling's theorem, without qualification. LK refer to p. 48, where a 'hydromagnet' is discussed; there it is explicitly said "the device cannot be self-exciting".
- Fearn et al. (1988) highlight the need for a mathematically more satisfactory proof than provided by Cowling, but they do not suggest that the theorem may actually be wrong. Braginskii (1965) provides a more rigorous proof (for the case $\nabla.\mathbf{v} = 0$); see also Backus & Chandrasekhar (1956).
- Alexeff (1989) starts by considering the case where $\mathbf{j} = \sigma\mathbf{v} \times \mathbf{B}$ (i.e. no $\sigma\mathbf{E}$ term), and then proceeds to derive an expression for the conductivity $\sigma \propto T^{3/2}$, the inclusion of which in a term $\sigma\mathbf{E}$ is then claimed to invalidate the theorem. This term was, of course, included in the original theorem. Thus Alexeff's argument adds nothing that is new.
- Lorrain's (1991) claim to demonstrate the failure of Cowling's theorem rests on a false assumption about the possible consistent relative configurations of magnetic and velocity fields in a conducting fluid. His counterexample is actually explicitly nonaxisymmetric, due to the introduction of baffles in meridian planes!
- Ingraham (1995) claims that the presence of discontinuities of magnetic fields at sharp physical boundaries renders invalid the underlying theorems on partial differential equations on which the Cowling theorem rests, although he does not produce a counter example. However, astrophysical bodies do not possess strictly sharp boundaries, and the magnetic field can be expected to be continuous everywhere.

Thus, none of the references cited by LK appears to contain any substantive evidence for the invalidity of Cowling's theorem. We should also mention at this point a number of other papers extending anti-dynamo theorems to more general configurations (eg Hide & Palmer 1982, Ivers & James 1986, 1988). We note also that, whilst Cowling's theorem can not be extended to more general physical circumstances, such as curved spacetime near Kerr black holes (Khanna & Camenzind 1996), valid counter examples have yet to be provided; see Brandenburg (1996) for the case of Kerr black holes.

## 2. The sunspot model

LK present what is claimed to be a stationary axisymmetric dynamo model for a sunspot. This has three salient features: the fluid is perfectly conducting, there is zero toroidal field, and the (steady) fluid flow is wholly in meridian planes. Unfortunately, LK do not state the boundary conditions on the velocity or magnetic field in their model, so it is unclear for example whether the velocity streamlines all close within the dynamo region, or if some extend beyond it. The following general statements can be made. In a perfect conductor, magnetic field lines or flux tubes are frozen into the fluid, and the magnetic flux in each tube is constant. In a steady flow with closed streamlines, the magnetic flux tubes are advected around the streamlines. Magnetic flux then cannot be created but, if **v** and **B** are not parallel, the field strength can be increased locally by field line stretching – at the expense of an ever decreasing length scale across the field.

(Eventually, of course, the field scale would become so small that the perfect conductivity assumption was invalid.) In a linear dynamo (no feedback of the magnetic field onto the fluid motions) with a steady velocity field, the magnetic flux must increase exponentially if the dynamo is excited. The field eventually adopts the form of the fastest growing eigenfunction, and *the spatial structure does not subsequently vary with time*.

If the streamlines are 'open', passing to the exterior of the dynamo region, there is the possibility that magnetic flux can be advected into the dynamo region from a source at large distances, more rapidly than it is advected outwards away from it. *However such advection of flux does not constitute dynamo action*: here flux is being transported, not created. Another possibility is that the boundary conditions may be such as to maintain a non-decaying magnetic flux on certain of the boundaries, and in that case the field strength within the region studied may be amplified (eg Weiss 1966) – again this is not dynamo action. A self-excited dynamo must be consistent with the condition that $\mathbf{B} \to \mathbf{0}$ at large distances.

Of course, with the assumption of perfect conductivity (as LK), the magnetic flux cannot decay, although it may be advected out from the region studied.

In fact, a simpler objection can be raised against LK's specific 'demonstration' of dynamo action. They examine the axisymmetric equation $\partial\mathbf{B}/\partial t = \nabla\times(\mathbf{v}\times\mathbf{B})$, in the case where $\partial\mathbf{B}/\partial t = \mathbf{0}$, and $\mathbf{v}$ and $\mathbf{B}$ lie in meridian planes, and claim that in their model the product $\mathbf{v}\times\mathbf{B}$ is in such a sense as to drive the (steady) dynamo. However, with their assumptions, $\mathbf{v}$ and $\mathbf{B}$ are necessarily parallel vectors!

Later in their paper LK introduce a finite value of the conductivity. But then Cowling's theorem certainly applies, for any appropriate choice of boundary conditions.

Subsequently, Lorrain (private communication) claims that setting the space charge density to zero introduces a fundamental error in standard MHD and so dynamo theory. However this is just the assumption made in LK. In any case, such a situation would mean that the assumptions of Cowling's theorem were there invalid, and hence that the theorem was inapplicable, not that the theorem was false. Moreover, situations in which charge separation occurs and contributes to magnetic field generation are recognized – this is the 'battery effect' (e.g. Biermann 1950, Mestel & Roxburgh 1962, Dolginov 1977).

## 3. Conclusions

In situations of astrophysical interest there are, of course, now many examples of fully three-dimensional dynamo action, for both laminar as well as turbulent flows. Note also that an axisymmetric laminar velocity field can drive a steady *nonaxisymmetric* dynamo (e.g. Gailitis 1970).

Nevertheless, we conclude that Lorrain & Koutchmy's (1998) claims, that Cowling's (1934) theorem is inapplicable in simple astrophysical situations, and to have found a self-excited axisymmetric dynamo operating in a sunspot model, are false. Cowling's theorem is alive and well!

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