HIGH-RESOLUTION SIMULATIONS OF NONHELICAL MHD TURBULENCE

N.E.L. HAUGEN[1], A. BRANDENBURG[2] and W. DOBLER[3]

[1]*Department of Physics, The Norwegian University of Science and Technology, Høyskoleringen 5, N-7034 Trondheim, Norway; E-mail: nils.haugen@phys.ntnu.no*

[2]*NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*

[3]*Kiepenheuer-Institut für Sonnenphysik, Schöneckstraße 6, D-79104 Freiburg, Germany*

Abstract. According to the kinematic theory of nonhelical dynamo action, the magnetic energy spectrum increases with wavenumber and peaks at the resistive cutoff wavenumber. It has previously been argued that even in the dynamical case, the magnetic energy peaks at the resistive scale. Using high resolution simulations (up to 1024^3 meshpoints) with no large-scale imposed field, we show that the magnetic energy peaks at a wavenumber that is independent of the magnetic Reynolds number and about five times larger than the forcing wavenumber. Throughout the inertial range, the spectral magnetic energy exceeds the kinetic energy by a factor of two to three. Both spectra are approximately parallel. The total energy spectrum seems to be close to $k^{-3/2}$, but there is a strong bottleneck effect and we suggest that the asymptotic spectrum is instead $k^{-5/3}$. This is supported by the value of the second-order structure function exponent that is found to be $\zeta_2 = 0.70$, suggesting a $k^{-1.70}$ spectrum. The third-order structure function scaling exponent is very close to unity,—in agreement with Goldreich–Sridhar theory.

Adding an imposed field tends to suppress the small-scale magnetic field. We find that at large scales the magnetic energy spectrum then follows a k^{-1} slope. When the strength of the imposed field is of the same order as the dynamo generated field, we find almost equipartition between the magnetic and kinetic energy spectra.

Keywords: interstellar medium, turbulence

1. Introduction

Magnetic fields may play an important role during star formation. Stars are generally formed in strongly magnetized regions, and the magnetic pressure that builds up in shocks and the initial collapse is likely to determine the detailed evolution.

Early simulations of hydromagnetic turbulence have always suggested that the magnetic field is more intermittent than the velocity field, if the field is generated by dynamo action (Meneguzzi et al., 1981; Kida et al., 1991). Furthermore, linear theory (Kazantsev, 1968) suggests that the magnetic spectrum should peak at the resistive scale, and it has been argued that this may hold even in the nonlinear regime (Maron and Blackman, 2002).

On the other hand, if there is an imposed large scale field, there is no doubt that most of the magnetic energy resides at large scales e.g., Cho and Vishniac (2000). The obvious question is therefore, are the cases of dynamo-generated and imposed fields really drastically different?

Astrophysics and Space Science **292:** 53–60, 2004.

The purpose of this paper is to compare the case of an imposed field with that of a dynamo-generated one. We begin by briefly reviewing the main results of our recent paper (Haugen et al., 2003), where we show that in the dynamo case the magnetic energy does *not* peak at the resistive scale.

2. Equations

Our approach is the same as in Haugen et al. (2003), which is similar to that of Brandenburg (2001), except that the flow is now forced without helicity. We adopt an isothermal equation of state with a constant sound speed $c_{\rm s}$, so the pressure p is related to the density ρ by $p = \rho c_{\rm s}^2$. The equation of motion is written in the form,

$$\frac{{\rm D}\boldsymbol{u}}{{\rm D}t} = -c_{\rm s}^2 \boldsymbol{\nabla} \ln \rho + \frac{\boldsymbol{J} \times \boldsymbol{B}}{\rho} + \boldsymbol{F}_{\rm visc} + \boldsymbol{f}, \tag{1}$$

where ${\rm D}/{\rm D}t = \partial/\partial t + \boldsymbol{u} \cdot \boldsymbol{\nabla}$ is the advective derivative, $\boldsymbol{J} = \boldsymbol{\nabla} \times \boldsymbol{B}/\mu_0$ is the current density, μ_0 is the vacuum permeability, $\boldsymbol{F}_{\rm visc}$ is the viscous force, and $\boldsymbol{f}$ is a random forcing function that consists of nonhelical plane waves. The continuity equation is written in terms of the logarithmic density,

$$\frac{{\rm D} \ln \rho}{{\rm D}t} = -\boldsymbol{\nabla} \cdot \boldsymbol{u}, \tag{2}$$

and the induction equation is solved in terms of the magnetic vector potential $\boldsymbol{A}$, where $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$, so

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{u} \times \boldsymbol{B} + \eta \boldsymbol{\nabla}^2 \boldsymbol{A}, \tag{3}$$

where $\eta = $ const is the magnetic diffusivity. We use periodic boundary conditions in all three directions for all variables.

The solutions are characterized by the kinetic and magnetic Reynolds numbers that are based on the forcing wavenumber $k_{\rm f}$ and defined as

$${\rm Re} = u_{\rm rms}/(\nu k_{\rm f}), \quad R_{\rm m} = u_{\rm rms}/(\eta k_{\rm f}), \tag{4}$$

respectively. The ratio of the two is the magnetic Prandtl number,

$$P_{\rm m} = \nu/\eta = R_{\rm m}/{\rm Re}. \tag{5}$$

We use the Pencil Code[1], which is a cache and memory efficient high-order finite-difference code (sixth order in space and third order in time) for solving the compressible MHD equations.

[1] http://www.nordita.dk/data/brandenb/pencil-code.

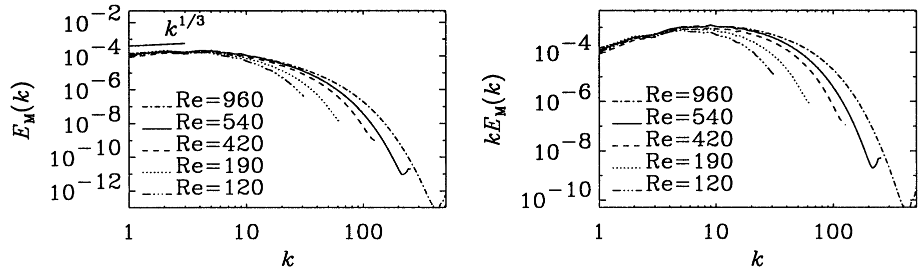

Figure 1. Left: Magnetic energy spectra for all our runs. We see that the peak of the magnetic energy spectrum is at $k = 5$ for all values of Re. *Right*: The peak of $kE_M(k)$ is found at $k \approx 9$ for large values of Re.

3. Results

3.1. The peak of the magnetic energy spectrum

We have run simulations with up to 1024^3 meshpoints to show that the magnetic energy spectrum does not peak at the resistive scale, as has previously been claimed (Maron and Blackman, 2002). In the left panel of Figure 1 we see that the peak of the spectrum is around $k = 5$ for all our runs, i.e., it is independent of Re. We also note that we find a $k^{1/3}$ slope for small values of k (Batchelor, 1950).

A more stringent measure is to look at the magnetic energy per unit logarithmic wavenumber interval, $kE_M(k)$, which would be flat if the contribution from small and large wavenumbers was equal. This is shown in the right hand panel of Figure 1. We see that the peak of $kE_M(k)$ is shifted toward smaller scales compared to $E_M(k)$, but it is still not at the resistive scale. We do indeed see that for the largest runs it seems to settle at $k \approx 10$, which is well within the inertial range.

3.2. The inertial range

In Figure 2 we see that there seems to be a clear inertial range for $7 \lesssim k \lesssim 25$, where $E_M(k)$ and $E_K(k)$ are parallel and have a slope of $k^{-3/2}$. The $k^{-3/2}$ slope is suggestive of the Iroshnikov (1963) and Kraichnan (1965) (IK) theory, and may seem incompatible with the Goldreich and Sridhar (1995) (GS) theory. We also note that in the inertial range, the relative fractions of magnetic and kinetic energy seem to be saturated at $E_M(k)/E_K(k) \approx 2.3$.

Knowing that IK theory predicts that the fourth-order structure function scales linearly ($\zeta_4 = 1$), while GS theory predicts linear scaling for the third-order structure function ($\zeta_3 = 1$), we now calculate the logarithmic derivatives for these structure functions; see Figure 3. From these plots we see that the IK theory cannot be correct since the fourth-order structure function is clearly steeper than linear (i.e., $\zeta_4 = 1.3 \neq 1$). The third-order structure function on the other hand scales linearly.

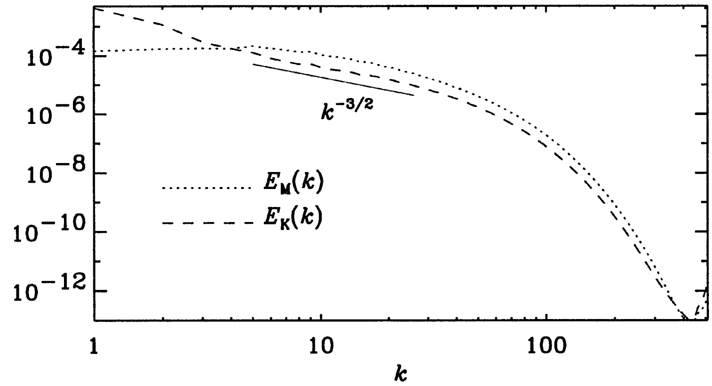

Figure 2. Kinetic and magnetic energy spectra of our largest run; 1024^3 mesh points.

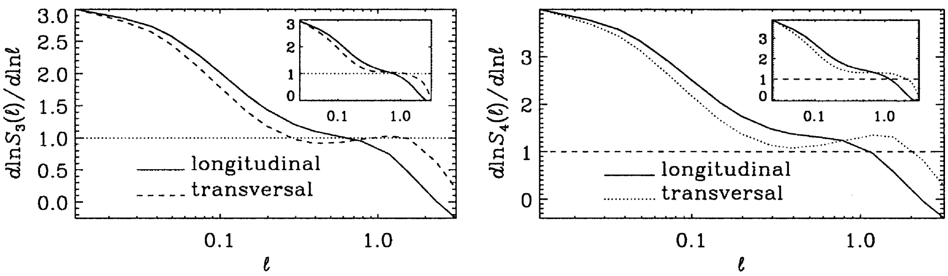

Figure 3. Left: logarithmic derivative of third-order structure function. The inset is for a run with 256^3 mesh points, while the large plot is for a run with 512^3 mesh points. The result is consistent with $\zeta_3 = 1$ *Right:* logarithmic derivative of fourth-order structure function. We see that this is clearly *not* compatible with IK theory, which would predict $\zeta_4 = 1$.

The second-order structure function scaling exponent of the Elsasser variable, $z^{\pm} = \boldsymbol{u} \pm \boldsymbol{B}/\sqrt{\rho\mu}$, is also indicative of GS theory being applicable, since we find $\zeta_2 = 0.7$, which implies $E_T(k) = E_M(k) + E_K(k) \propto k^{-(1+\zeta_2)} = k^{-1.7}$.

As we have argued earlier (Haugen et al., 2003), the reason that Figure 2 shows a $k^{-3/2}$ inertial range is that there is a strong bottleneck effect (Falkovich, 1994). It turns out that this bottleneck is much stronger in three-dimensional than in one-dimensional energy spectra (Dobler et al., 2003). We therefore plot the one-dimensional counterpart of Figure 2 in Figure 4. Here we see that the inertial range indeed has a slope close to $-5/3$, as suggested by our previous findings from the structure functions. From this we conclude that also the three-dimensional energy spectra will show a $k^{-5/3}$ inertial range far enough away from the diffusive subrange.

3.3. Imposed magnetic field

Until now, we have only been looking at the case with no externally imposed field. We, therefore, want to see what effect such imposed fields of varying strengths

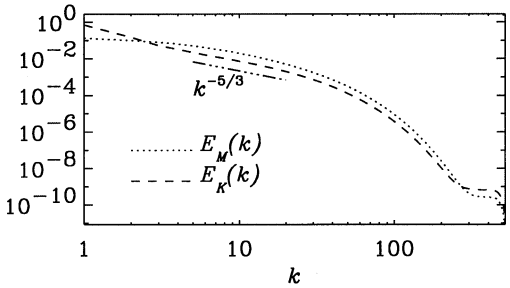

Figure 4. One-dimensional energy spectra of our largest run. We see here that the inertial range is consistent with a $k^{-5/3}$ slope.

have on the dynamo. In Figure 5, we plot energy spectra for simulations with 128^3 meshpoints (left panel) and 256^3 meshpoints (right panel) for different imposed fields. From the figure we see that having an imposed field seems to increase the large-scale magnetic field and at the same time *decreases* the small-scale field. The stronger the external field, the stronger the suppression of the small-scale fields. The same effect is clearly seen in Figure 7. We also note that with an imposed field, the peak of the magnetic energy is found at the largest scale ($k = 1$), not at $k = 5$ as in the case without an imposed field. Looking at Figure 6 we see that for $1 < k < 4$ there seems to be a k^{-1} slope for $E_M(k)$. Such a slope for the large-scale magnetic field has been suggested previously (Ruzmaikin and Shukurov, 1982; Kleeorin and Rogachevskii, 1994; Brandenburg et al., 1996; Matthaeus and Goldstein, 1986). From $k \approx 6$ there seems to be a short inertial range with a $k^{-5/3}$ slope, as expected from GS theory.

When the strength of the imposed field is comparable to the dynamo generated field ($B_0 = 0.06$ for 128^3 meshpoints, and $B_0 = 0.3$ for 256^3 meshpoints in Figure 5), we see that there is almost equipartition between magnetic and kinetic energy spectra, at least for the smaller scales. On the other hand, Cho and Vishniac (2000) find almost perfect equipartition for all scales. The difference is small, and could perhaps be explained by a difference in the forcing function. We did check, however, that changing from a delta-correlated forcing function to one with a renewal time comparable to the turnover time does not resolve this relatively minor discrepancy.

4. Conclusions

Our simulations suggest that both magnetic and kinetic energy spectra show power-law behavior and are proportional to $k^{-5/3}$. The peaks of these spectra are therefore located at relatively small wavenumbers; $k = 5$ and $k = k_f = 1$, respectively. When applied to the interstellar medium, where the forcing wavenumber corresponds to a scale of up to 100 pc, the scale of the magnetic peak would be at a wavenumber

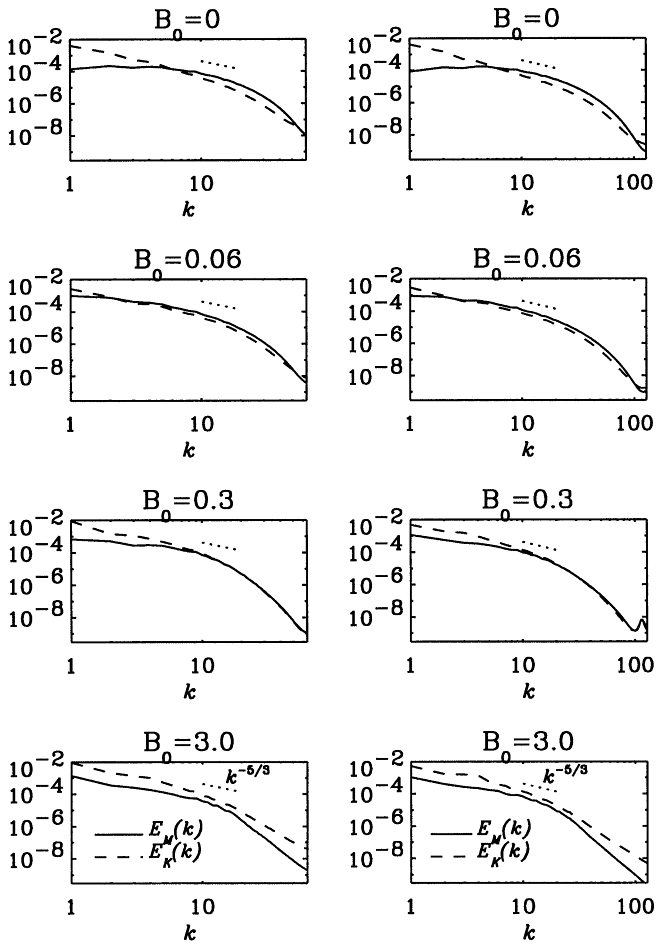

Figure 5. Energy spectra for simulations with imposed fields of different strengths. Solid lines show magnetic spectra; dashed lines show kinetic spectra. *Left:* resolution 128^3 meshpoints; Reynolds numbers vary between 200 and 400. *Right:* 256^3 meshpoints; Reynolds numbers vary between 320 and 500. In both cases the large Reynolds numbers correspond to the cases with $B_0 = 0.3$ and $B_0 = 3.0$, where the kinetic motions are least suppressed by the magnetic fields.

corresponding to a scale of about 10 pc. Thus, the peak is not anywhere near the resistive scale, as has previously been suggested. There is of course still energy at scales as small as the resistive scale ($\sim 10^8$ cm), as evidenced by interstellar scintillation measurements, but this energy can only be a small fraction of the total magnetic energy.

When a magnetic field is imposed, several features change, the magnetic spectrum peaks at $k = 1$, but at higher wavenumbers the magnetic energy is suppressed compared to the case without an imposed field. We believe this is because of a suppression of dynamo action. This highlights once again that imposed fields are

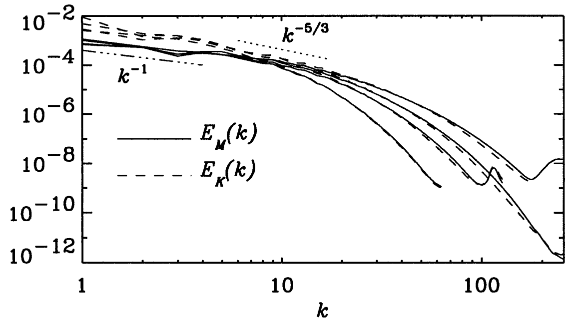

Figure 6. Convergence of energy spectra for runs with increasing Re (=280, 500, 650, and 1170), and with $B_0 = 0.3$. We see a k^{-1} for $1 < k < 4$, and we also start to see the appearance of a short $k^{-5/3}$ inertial range beginning at $k \approx 8$.

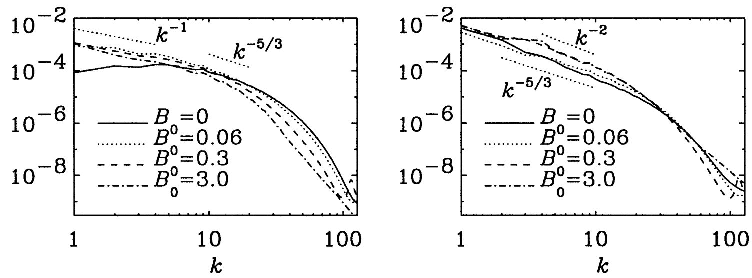

Figure 7. Power spectra for runs with different imposed fields; Re = 320–500. *Left:* magnetic energy spectra; we see that the the larger the external field the more the magnetic field at small scales is suppressed. *Right:* kinetic energy spectra.

somewhat artificial and do not represent the case of a self-generated large-scale magnetic field in a small subvolume of a large turbulent dynamo system.

Acknowledgements

Use of the parallel computers in Trondheim (Gridur), Odense (Horseshoe), Leicester (Ukaff), and Bergen (Fire) is acknowledged.

References

Batchelor, G.K.: 1950, *Proc. Roy. Soc. Lond.* **A201**, 405.
Brandenburg, A.: 2001, *ApJ* **550**, 824.
Brandenburg, A., Jennings, R.L., Nordlund, Å., Rieutord, M., Stein, R.F. and Tuominen, I.: 1996, *J. Fluid Mech.* **306**, 325.
Cho, J. and Vishniac, E.: 2000, *ApJ* **539**, 273.

Dobler, W., Haugen, N.E.L., Yousef, T.A. and Brandenburg, A.: (2003), *Phys. Rev. E* **68**, 026304.
Falkovich, G.: 1994, *Phys. Fluids* **6**, 1411.
Goldreich, P. and Sridhar, S.: 1995, *ApJ* **438**, 763.
Haugen, N.E.L., Brandenburg, A. and Dobler, W.: (2003), *ApJ* **597**, L141.
Iroshnikov, R.S.: (1963), *Sov. Astron.* **7**, 566.
Kazantsev, A.P.: (1968), *Sov. Phys. JETP* **26**, 1031.
Kida, S., Yanase, S. and Mizushima, J.: 1991, *Phys. Fluids* **A3**, 457.
Kleeorin, N. and Rogachevskii, I.: (1994), *Phys. Rev.* **E 50**, 2716.
Kraichnan, R.H.: 1965, *Phys. Fluids* **8**, 1385.
Maron, J. and Blackman, E.G.: (2002), *ApJ* **566**, L41.
Matthaeus, W.H. and Goldstein, M.L.: (1986), *Phys. Rev. Lett.* **57**, 495.
Meneguzzi, M., Frisch, U. and Pouquet, A.: (1981), *Phys. Rev. Lett.* **47**, 1060.
Ruzmaikin, A.A. and Shukurov, A.M.: (1982), *Astrophys. Spa. Sci.* **82**, 397.