Magnetic helicity in primordial and dynamo scenarios of galaxies

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Some common properties of helical magnetic fields in decaying and driven turbulence are discussed. These include mainly the inverse cascade that produces fields on progressively larger scales. Magnetic helicity also restricts the evolution of the large-scale field: the field decays less rapidly than a non-helical field, but it also saturates more slowly, i.e. on a resistive time scale if there are no magnetic helicity fluxes. The former effect is utilized in primordial field scenarios, while the latter is important for successfully explaining astrophysical dynamos that saturate faster than resistively. Dynamo action is argued to be important not only in the galactic dynamo, but also in accretion discs in active galactic nuclei and around protostars, both of which contribute to producing a strong enough seed magnetic field. Although primordial magnetic fields may be too weak to compete with these astrophysical mechanisms, such fields could perhaps still be important in producing polarization effects in the cosmic background radiation.

1 Introduction

Magnetic helicity plays a fundamental role both in primordial theories of galactic magnetism as well as in dynamo theories amplifying and sustaining contemporary galactic fields. Both issues have been reviewed in recent years (Grasso & Rubinstein 2001; Widrow 2002; Giovannini 2004; Brandenburg & Subramanian 2005a). We will therefore only try to collect the main points relevant to the issues concerning magnetic helicity in galactic and protogalactic magnetism.

The main reason magnetic helicity is at all of concern to us is that even in the resistive case the rate of magnetic helicity dissipation asymptotes to zero as the magnetic Reynolds number goes to infinity. This is not the case with magnetic energy dissipation, which remains always important, and does not decrease with increasing magnetic Reynolds number (Galsgaard & Nordlund 1996). Therefore the magnetic helicity is nearly conserved at all times. This has serious consequences for the evolution of magnetohydrodynamic (MHD) turbulence, as has been demonstrated by a number of recent simulations when the resolution has been large enough (Brandenburg 2001a; Mininni et al. 2005).

At a more descriptive level, magnetic helicity characterizes the degree of field line linkage. As the magnetic field relaxes, its energy decreases, but the linkage stays, at least as much as possible. The field's inability to undo its knots implies also that the field cannot decay freely. This slows down the decay, which is important if a primordial field is to be of any significance at the time of recombination. In the driven case, on the other hand, magnetic helicity is

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better pictured in terms of writhe and twist (e.g. Longcope & Klapper 1997; Démoulin et al. 2002). Writhe refers to the tilt of a flux tube, and we use both expressions synonymously. A cyclonic event tilts individual flux tubes, but as it does so, a corresponding amount of internal twist is necessarily introduced in the tube (Blackman & Brandenburg 2003). This is what saturates the dynamo, and this can be a very powerful effect if the small-scale internal twist cannot escape. In this review we discuss both decaying and driven turbulence. The former is relevant for prolonging the decay of a primordial field, while the latter is relevant for understanding how the galactic dynamo saturates and how to enable it to do so faster.

2 Magnetic helicity in the primordial scenario

Theories of the electroweak phase transition, about 10^{-10} s or less after the big bang, allow for the possibility of generating a magnetic field of up to 10^{24} G (see Grasso & Rubinstein 2001). [In practice the field will be weaker; Brandenburg et al. (1996a) discussed a field of 10^{18} G at the time of the electroweak phase transition which would have decayed to 10^{-11} G at the present time.] The scale of this field would be less than or comparable to the horizon scale which was only about $3\,\text{cm}$ or less. With the cosmological expansion this field would have a scale of about $1\,\text{AU}$, which is still small compared with the scale of galaxies (Hindmarsh & Everett 1998). This led to the idea that the inverse cascade of magnetic helicity might have played a role in increasing the scale of the turbulent magnetic field (Brandenburg et al. 1996a; Field & Carroll 2000); see also Brandenburg

(2001b) for a summarizing view. Therefore we address in this section how a helical magnetic field decays. The only source of turbulence is assumed to be the initial magnetic field itself, which drives a flow through the Lorentz force.

2.1 Scaling of the energy spectrum during inverse transfer

There are indeed certain possibilities for producing primordial magnetic fields that may have had significant amounts of magnetic helicity (Joyce & Shaposhnikov 1997; Cornwall 1997; Vachaspati 2001; Semikoz & Sokoloff 2004, 2005). Letting the field inverse cascade has also the advantageous side effect that the resulting large-scale fields can more easily overcome Silk damping during the period of recombination (Brandenburg et al. 1997). This damping was previously thought to be a serious threat to primordial theories that generated magnetic field during early Universe phase transitions, but calculations showed that the Alfvén mode can survive for scales smaller than the Silk scale (Subramanian & Barrow 1998a; Jedamzik et al. 1998).

More recently simulations have directly been able to demonstrate how the inverse cascade works. This can be seen from magnetic energy spectra at different times after initializing the simulation with a random helical magnetic field. Figure 1 demonstrates quite clearly that in decaying turbulence an inverse cascade means not just that the dominant scale increases, because any diffusion that is more efficient on smaller scales than on larger scales must increase the relative dominance of large-scale fields over small-scale fields. Instead, inverse cascade means actually a real increase of the field strength at large scales, i.e. the spectral energy $E(k,t)$ increases with t for $k < k_{peak}$. Here, k_{peak} is the wavenumber where the magnetic power spectrum peaks; this value is decreasing with time.

The work of Christensson et al. (2001) has also shown that the spectrum stays approximately shape-invariant; see also Banerjee & Jedamzik (2004). Indeed, the time-dependent spectrum can be written as

$$E_M(k,t) = \xi(t)^{-q} g_M(k\xi), \tag{1}$$

where $\xi(t)$ is the characteristic length scale of the magnetic field and $g_M(k\xi)$ is the magnetic scaling function [see Christensson et al. (2001) for a plot of $g_M(k\xi)$]. The magnetic energy can then be written as

$$E_M(t) = \int_0^{k_{max}} E_M(k,t)\, dk \propto \xi^{-(q+1)}. \tag{2}$$

Here we have assumed that the spectrum has an upper cutoff at k_{max}, which is comparable to and probably slightly larger than k_{peak}. Furthermore, if the characteristic length scale changes with time in a power law fashion, $\xi \propto t^r$, we find a decay law of magnetic energy like $E_M(t) \propto t^n$ with $n = r(q+1)$. This allows us now to calculate how the spectral energy at large scales (small values of k) depends on time. For small values of k we now assume that Eq. (1) can be written in power law form as

$$E_M(k,t) = k^p t^\sigma \quad \text{(for } k \ll k_{max}\text{)}. \tag{3}$$
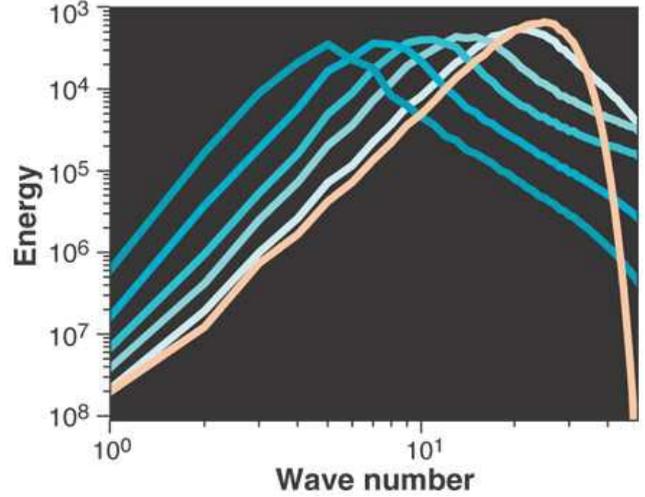

Fig. 1 Magnetic energy spectra at different times (increasing roughly by a factor of 2). The curve with the right-most location of the peak corresponds to the initial time, while the other lines refer to later times (increasing from right to left). Note the propagation of spectral energy to successively smaller wavenumbers k, i.e. to successively larger scales. Adapted from Christensson et al. (2001) and Brandenburg (2001b).

We are interested in the exponent σ that tells us how the spectral magnetic energy grows in time. Using Eq. (1), assuming that $g_M(k\xi) = (k\xi)^p$, we have

$$E_M(k,t) = \xi(t)^{-q}(k\xi)^p = k^p \xi^{p-q} = k^p t^{r(p-q)}. \tag{4}$$

Expressing q in terms of n, we find

$$\sigma = r(p+1) - n. \tag{5}$$

In Table 1 we give the results for different values of p, n, and r. The first entry in this table ($p = 4$, $n = 1/2$, and $r = 1/2$) is basically the case considered by Christensson et al. (2005), except that they also found an additive correction to n (see below), which directly affects σ.

Table 1 Values of σ for different combinations of p, n, and r, as given by Eq. (5). The first row applies to helical turbulence in the limit of large magnetic Reynolds numbers with $p = 4$.

p	n	r	σ
4	1/2	1/2	2
2	1/2	1/2	1
0	1/2	1/2	0
4	1	1/2	3/2
2	1	1/2	1/2
p	n	0	$-n$
4	1	1	4

Looking at Table 1 and also at Eq. (5), it is clear that a steeper spectrum (larger p) and a faster increase of the length scale (larger r) yield a faster rise of the spectral

power at low $k < k_{\mathrm{max}}$, while the overall decay exponent, $-n$, directly adds to σ. For the case considered by Christensson et al. (2001), where $p = 4$, $n \approx 0.7$, $r \approx 0.5$, one finds $\sigma \approx 1.8$, which is compatible with the rise of spectral energy (for $k < k_{\mathrm{peak}}$) seen in Fig. 1.

2.2 Simple argument for inverse transfer

At this point it may be useful to provide a simple argument [due to Frisch et al. (1975)] as to why the interaction of helical magnetic fields leads preferentially to large-scale magnetic fields. We reproduce here the argument as presented in the review by Brandenburg & Subramanian (2005a). In this argument one assumes that two waves with wavevectors $\boldsymbol{p}$ and $\boldsymbol{q}$ interact with each other to produce a wave of wavevector $\boldsymbol{k}$. Both waves are assumed to be fully helical with the same sign of helicity. Assuming that the total energy E (which is the sum of magnetic and kinetic energies) is conserved together with magnetic helicity, we have

$$E_p + E_q = E_k, \tag{6}$$

$$|H_p| + |H_q| = |H_k|. \tag{7}$$

(Since in this system the flow is driven by the magnetic field, we can ignore the kinetic energy compared with the magnetic energy, so for all practical purposes we can think of E being equivalent to E_{M}.) Since both waves are fully helical, we have

$$2E_p = p|H_p| \quad \text{and} \quad 2E_q = q|H_q|, \tag{8}$$

and so Eq. (6) yields

$$p|H_p| + q|H_q| = 2E_k \geq k|H_k|, \tag{9}$$

where the last inequality is also known as the realizability condition that is here applied to the target wavevector $\boldsymbol{k}$ after the interaction. Using Eq. (7) in Eq. (9) we have

$$p|H_p| + q|H_q| \geq k(|H_p| + |H_q|). \tag{10}$$

In other words, the target wavevector $\boldsymbol{k}$ after the interaction of wavevectors $\boldsymbol{p}$ and $\boldsymbol{q}$ satisfies

$$k \leq \frac{p|H_p| + q|H_q|}{|H_p| + |H_q|}. \tag{11}$$

The expression on the right hand side of Eq. (11) is a weighted mean of p and q and thus satisfies

$$\min(p, q) \leq \frac{p|H_p| + q|H_q|}{|H_p| + |H_q|} \leq \max(p, q), \tag{12}$$

and therefore

$$k \leq \max(p, q). \tag{13}$$

In the special case where $p = q$, we have $k \leq p = q$, so the target wavenumber after interaction is always less than or equal to the initial wavenumbers. In other words, wave interactions tend to transfer some magnetic energy to smaller wavenumbers, i.e. to larger scale. This corresponds to an inverse cascade. The realizability condition, $\frac{1}{2}k|H_k| \leq E_k$, was the most important ingredient in this argument. An important assumption that we made in the beginning was that the initial field be fully helical; see Maron & Blackman (2002) and Brandenburg et al. (2002) for simulations of driven turbulence with fractional helicity.

2.3 Decay law

The magnetic energy decay is often seen to follow power law behavior, i.e. $E(t) \sim t^{-n}$. For nonhelical turbulence, n is typically larger than unity [e.g. $n = 1.28$ in the work of Mac Low et al. (1998), or $n = 1.2$ in the argument discussed by Subramanian et al. (2006)]. On the other hand, for helical turbulence the decay is more shallow; for example Biskamp & Müller (1999) find typical values between 0.5 and 0.7. They explain their scaling with the following argument. They assume that the magnetic helicity H is perfectly conserved, so $H(t) = \mathrm{const}$, and so the typical length scale $L(t)$ depends only on the total energy, $E(t)$, via $L \sim H/E \sim E^{-1}$. Assuming furthermore that the rate of energy decay, ϵ, is proportional to U^3/L, where $U \sim E^{1/2}$ is the typical velocity, we have

$$-\frac{\mathrm{d}E}{\mathrm{d}t} \equiv \epsilon \sim \frac{U^3}{L} \sim \frac{E^{3/2}}{L} \sim E^{5/2}, \tag{14}$$

and integration over t yields

$$E \sim t^{-2/3}. \tag{15}$$

Although this decay law seems compatible with the numerical results within the range of magnetic Reynolds numbers they considered, its validity has been challenged on the grounds that $H(t)$ is not strictly conserved, but that it too must decay. Christensson et al. (2005) used the fact that $H(t)$ obeys the decay law

$$\dot{H} = -2\eta k_{\mathrm{d}}^2 H, \tag{16}$$

where $2\pi/k_{\mathrm{d}} \equiv \ell_{\mathrm{d}}$ is the typical scale on which magnetic helicity dissipation occurs. The decay law of $H(t)$ can only have power law behavior if k_{d} scales like $k_{\mathrm{d}} \sim t^{-1/2}$. We make use of this assumption and write this relationship in the following more explicit form:

$$k_{\mathrm{d}} = k_{\mathrm{d}0} \, (t/t_0)^{-1/2}, \tag{17}$$

where $k_{\mathrm{d}0}$ and t_0 are suitably defined constants. With this we have

$$H \sim t^{-2s}, \quad \text{where} \quad s = \eta k_{\mathrm{d}0}^2 t_0. \tag{18}$$

Simulations show that a number of different length scales, including ℓ_{d} and L, are all proportional to each other, and that their ratios are independent of time. Since $E = H/L$, we find that the energy decay law is

$$E \sim t^{-2s-1/2}. \tag{19}$$

The correction to the exponent, $2s$, vanishes in the limit of large magnetic Reynolds numbers, so that for all practical purposes the energy decay law is $E \sim t^{-1/2}$. The same scaling law, including the correction term for finite magnetic Reynolds numbers, has also been obtained by Campanelli (2004) using different scaling arguments. For comparison with simulations, however, the finite magnetic Reynolds number correction can be important. Empirically, Christensson et al. (2005) found that $s \approx 25/R_{\mathrm{m}}$.

We summarize this section by stressing once more the particular importance played by the magnetic helicity equation and, more specifically, the resistively slow evolution of

the magnetic helicity for large magnetic Reynolds numbers. Obviously, the magnetic helicity only plays a role if H is indeed finite and in fact large enough. The question "how large is large?" has not yet been addressed, because most studies assume the field to be maximally helical. This means that the magnetic helicity spectrum obeys $|kH(k)| = 2E_\mathrm{M}$, i.e. the realizability condition is saturated. However, even if the fractional magnetic helicity is initially small, because E tends to decay faster than H, the fractional magnetic helicity will gradually increase (Vachaspati 2001).

A more serious problem is whether significant levels of magnetic field strengths can be generated. The general consensus is now that it may be difficult, albeit not impossible, to have still a field strength of around 10^{-9} G at the present time. Such a field might have led to measurable polarization in the cosmic microwave background (Subramanian & Barrow 1998b, 2002; Seshadri & Subramanian 2001; Mack et al. 2002; Lewis 2004). It may also be possible to detect the presence of magnetic helicity through the production of a parity-odd component of gravity waves, which induces parity-odd polarization signals (Caprini et al. 2004; Kahniashvili & Ratra 2005). A 10^{-9} G field would also provide a sufficiently powerful seed magnetic field for explaining the generation and maintenance of fields with equipartition field strength. This will be discussed in the next section.

3 Magnetic helicity in dynamos

Before we focus specifically on the importance of magnetic helicity in dynamos, we discuss first whether in the dynamo scenario a significant magnetic field strength can be generated. The overall problem lies in the fact that the time scale, on which a global ordered magnetic field on the scale of galaxies can be generated, is likely to be comparable to the age of galaxies. To be successful, one has to have a strong enough seed magnetic field (Rees 1987). Typical e-folding times are on the order of the rotation period, which is around 2×10^8 yr; see Beck et al. (1996). Such times may be too long in view of the fact that in some very young high redshift galaxies (age 10^9 yr) typical field strengths are already in the microgauss range (Kronberg et al. 1992; Perry et al. 1993; Kronberg 1994). Within a time as short as 5 e-folding times one would only be able to amplify the field by a factor of 150.

3.1 Outflows from AGNs or YSOs for seeding galaxies

In addition to primordial magnetic fields, a potentially much stronger source of seed magnetic fields might be provided by stellar winds and the outflows of protostellar discs around young stellar objects (YSOs), as well as discs around active galactic nuclei (AGNs). AGNs might provide more coherent fields because their scale is larger. The general idea of outflows seeding the interstellar medium has been around for some time (see, e.g., Goldshmidt & Rephaeli

1993, 1994; Völk & Atoyan 1999; Brandenburg 2000; Kronberg et al. 2001). On the average the coherence scale of the field in clusters of galaxies is $5\,\mathrm{kpc}$ (Clarke et al. 2001), but locally it can be much larger. Along some of the well developed radio lobes the coherence scale can be as big as a megaparsec (Govoni et al. 2001).

In order to estimate the resulting field strength, let us here reproduce an earlier estimate by Brandenburg (2000). The basic idea is that outflows (both on stellar and on galactic scales) tend to be magnetized. Their power or kinetic luminosity is roughly

$$L_\mathrm{kin} \approx \dot{M}_\mathrm{w} c_\mathrm{s}^2, \tag{20}$$

where $\dot{M}_\mathrm{w}$ is the mass loss rate into the wind and c_s is the sound speed of the ambient gas. The outflow speed tends to be a certain multiple of this. Assuming that the ratio of magnetic to kinetic luminosities, $L_\mathrm{mag}/L_\mathrm{kin}$, is constant [about 0.05 in the work of von Rekowski et al. (2003)] we can estimate the mean injection of magnetic fields to a cluster with N sources, each working over a time span Δt, distributed over a total volume L^3. This gives a magnetic energy of

$$E_\mathrm{mag} = N L_\mathrm{mag} \Delta t \tag{21}$$

for the entire cluster, and a root-mean-square field strength of

$$\langle \boldsymbol{B}^2 \rangle^{1/2} = \left(8\pi E_\mathrm{mag}/L^3 \right)^{1/2} \tag{22}$$

in cgs units.

Assuming $\dot{M}_\mathrm{w} = 0.1\,\mathrm{M}_\odot\,\mathrm{yr}^{-1} \approx 10^{25}\,\mathrm{g\,s}^{-1}$ for an AGN disc, $c_\mathrm{s} = 1000\,\mathrm{km\,s}^{-1}$ for a galaxy cluster, we have $L_\mathrm{kin} \approx 10^{41}\,\mathrm{erg\,s}^{-1}$, and hence $L_\mathrm{mag} \approx 10^{39}...10^{40}\,\mathrm{erg\,s}^{-1}$. Assuming $\Delta t = 0.1\,\mathrm{Gyr}$ and $N = 10^4$ we have $E_\mathrm{mag} \approx 10^{59}\,\mathrm{erg}$ for the entire cluster. Thus, $\langle \boldsymbol{B}^2 \rangle^{1/2} \approx 0.3\,\mu\mathrm{G}$.

For stellar winds and young stellar objects (YSOs) we obtain a very similar estimate. Assuming $\dot{M}_\mathrm{w} = 10^{-8}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1} \approx 10^{18}\,\mathrm{g\,s}^{-1}$ for a disc around a young stellar object, $c_\mathrm{s} = 10\,\mathrm{km\,s}^{-1}$ for the warm interstellar medium, we have $L_\mathrm{kin} \approx 10^{30}\,\mathrm{erg\,s}^{-1}$, and hence $L_\mathrm{mag} \approx 10^{28}...10^{29}\,\mathrm{erg\,s}^{-1}$. Assuming $\Delta t = 1\,\mathrm{Myr}$ and $N = 10^{11}$ we have $E_\mathrm{mag} \approx 10^{53}\,\mathrm{erg}$ for an entire galaxy. Thus, again, $\langle \boldsymbol{B}^2 \rangle^{1/2} \approx 1\,\mu\mathrm{G}$.

A potential problem with these approaches is that the magnetized winds may not actually be able to penetrate much (Jafelice & Opher 1992). A completely different idea is to produce strong enough seed magnetic fields in protogalactic turbulence by the small-scale dynamo, whose time scale is much shorter (10^7 yr); see Beck et al. (1994) for more details. The small-scale dynamo could produce a significant $\overline{\boldsymbol{u} \cdot \boldsymbol{b}}$ correlation which would contribute to the α effect (Yoshizawa & Yokoi 1993; Brandenburg & Urpin 1998). It is also possible that a combination of outflows together with small scale dynamo action might be providing the necessary seed for the large scale dynamo.
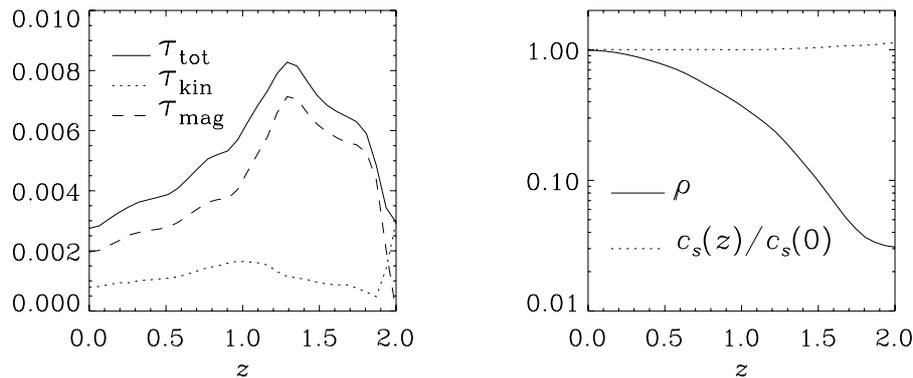
Fig. 2   Dependence of the stress component $\overline{\Pi}_{\varpi\phi}^{(SS)}$ (here denoted by τ_{xy}), separately for the kinetic and magnetic contributions, together with the sum of the two denoted by total (left) as well as the vertical dependence of density and sound speed (right). Note that τ_{xy} is neither proportional to the density ρ nor the sound speed $c_{\rm s}$. Adapted from Brandenburg et al. (1996b).

3.2   Disc corona heating by the MRI

In order to drive the outflows that may contribute to seeding the interstellar medium and that remove small-scale magnetic helicity from the dynamo (see next section), we need to discuss briefly the physics of disc coronae from where such outflows emerge.

A highly probable source of turbulence in any accretion disc is the Balbus & Hawley (1991) or magneto-rotational instability (MRI). Simulations show that the MRI together with the dynamo instability can produce a doubly-positive feedback, sustaining both the turbulence and the magnetic field necessary to drive the turbulence; see Brandenburg et al. (1995), Hawley et al. (1996), Stone et al. (1996). As has been emphasized in a number of papers, the MRI has the property of liberating most of its energy in the outer parts of the disc or rather the disc corona, where the density is low and the heating per unit mass therefore high. This was originally demonstrated only for nearly isothermal discs (Brandenburg et al. 1996b), see Fig. 2, but this has now also been shown for radiating discs (Turner 2004).

The mechanism of heating disc coronae described here is essential in the aforementioned picture of driving magnetized winds from accretion discs. It should however be noted that the conical outflows found by von Rekowski et al. (2003) may actually be more general and have now also been seen in fully three-dimensional simulations (De Villiers et al. 2005).

3.3   Importance of outflows for dynamos

Over the past 10–15 years it has become clear that the original mean field dynamo theory misses something important regarding its saturation properties. It started off by numerical calculations of the diffusion of a mean magnetic field in two dimensions (Cattaneo & Vainshtein 1991). These simulations indicated severe quenching of the turbulent magnetic diffusivity with increasing magnetic Reynolds number. This prompted similar investigations of the α effect (Vainshtein & Cattaneo 1992). Catastrophic quenching of the α effect was later confirmed using three-dimensional simulations (Cattaneo & Hughes 1996). Calculations involving magnetic helicity arguments were already presented by Gruzinov & Diamond (1994) and Bhattacharjee & Yuan (1995), confirming again a magnetic Reynolds number-dependent (i.e. catastrophic) α quenching. Another idea was that a sub-equipartition field would lead to the suppression of Lagrangian chaos (Cattaneo et al. 1996). The general idea was that small-scale magnetic fields grow rapidly to equipartition field strength, and that at this point the α effect shuts off (see also Kulsrud & Anderson 1992). Clearly, if mean field theory has anything to do with large-scale dynamos in galaxies or even the much smaller AGN and YSO discs, then something must be wrong with the idea of premature or catastrophic quenching (Field 1996).

Only over the past 5 years it became clear that the real culprit is indeed the magnetic helicity of the small-scale field, as was already suggested by Gruzinov & Diamond (1994, 1995, 1996) and Bhattacharjee & Yuan (1995), and that this problem might then be possible to solve by allowing for outflows of small-scale magnetic helicity through the boundaries (Blackman & Field 2000a,b; Kleeorin et al. 2000, 2002, 2003). The detailed quenching behavior seen in simulations (Brandenburg 2001a) was important in developing a revised mean field theory (Field & Blackman 2002; Blackman & Brandenburg 2002; Subramanian 2002). All these papers have in common an explicitly time-dependent equation for a magnetic contribution to the α effect. The resulting explicitly time-dependent equation is virtually identical to the old time-dependent quenching theory of Kleeorin & Ruzmaikin (1982).

A pictorial explanation of these new developments can be given as follows. Stratified rotating turbulence produces cyclonic motions, just as envisaged by Parker (1955). This produces in a systematic fashion a tilt in toroidal flux tubes as they rise owing to either thermal or magnetic buoyancy.
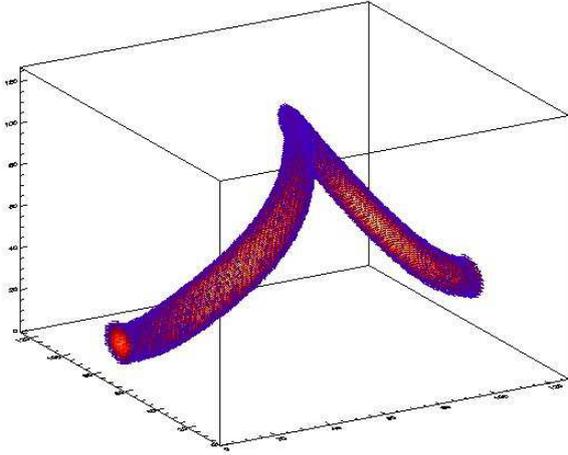
Fig. 3 Magnetic flux tube constructed from a Cauchy solution describing analytically the tilting and associated internal twisting of the tube. Adapted from Yousef & Brandenburg (2003).
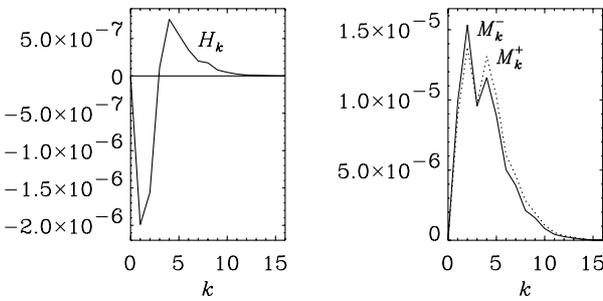

Fig. 5 Evolution of the energies of the total field $\langle \boldsymbol{B}^2 \rangle$ and of the mean field $\langle \overline{\boldsymbol{B}}^2 \rangle$, in units of B_{eq}^2, for runs with non-helical forcing and open or closed boundaries; see the solid and dotted lines, respectively. The inset shows a comparison of the ratio $\langle \overline{\boldsymbol{B}}^2 \rangle / \langle \boldsymbol{B}^2 \rangle$ for nonhelical ($\alpha = 0$) and helical ($\alpha > 0$) runs. For the nonhelical case the run with closed boundaries is also shown (dotted line near $\langle \overline{\boldsymbol{B}}^2 \rangle / \langle \boldsymbol{B}^2 \rangle \approx 0.07$). Note that saturation of the large-scale field occurs on a dynamical time scale; the resistive time scale is given on the upper abscissa. Adapted from Brandenburg (2005).

Fig. 4 Initial spectra of magnetic helicity, H_k, and of magnetic energy of positively and negatively polarized components, M_k^+ and M_k^-, respectively, for the tilted and twisted flux tube depicted in Fig. 3. Adapted from Yousef & Brandenburg (2003).

This tilt is the source of producing a poloidal field from a systematically oriented toroidal field. However, what was not included in this picture is the fact that an externally imposed tilt must necessarily yield an internal twist in the tube (Blackman & Brandenburg 2003). This can be seen in a semi-analytically generated Cauchy solution of an initially straight tube subject to a simple rising and twisting motion (Yousef & Brandenburg 2003); see Fig. 3. The magnetic helicity spectrum confirms a distinctively bi-helical behavior; see Fig. 4. This shows that due to this imposed motion no net magnetic helicity is produced, and that this is done in such a way that finite magnetic helicity is being produced with opposite signs at large scales ($H_k < 0$) and at smaller scales ($H_k > 0$). The same result was also obtained by Blackman & Brandenburg (2003), who calculated numerically the rise, expansion, and subsequent tilt of a flux tube in the presence of the Coriolis force.

The consequences of producing small-scale magnetic helicity can be dramatic in some cases (e.g. in periodic boxes). How this works has to do with another development that has its roots way in the past (Pouquet et al. 1976),
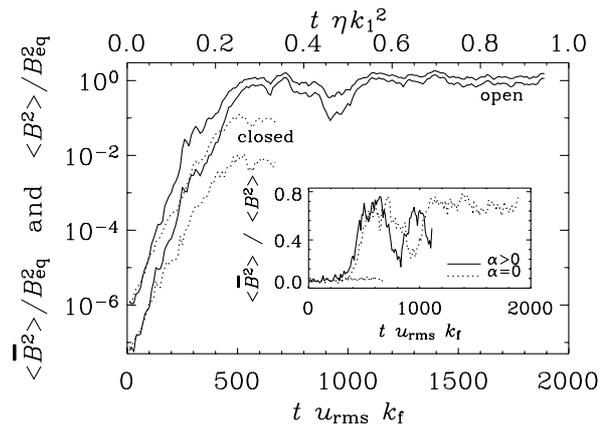
but whose consequences were not appreciated until more recently. The point is that all the analytically derived expressions for the α effect must be attenuated by an extra term (a magnetic α effect) that is proportional to the small-scale current helicity, $\overline{\boldsymbol{j} \cdot \boldsymbol{b}}$, in the isotropic case, or a corresponding modification proportional to $\epsilon_{ijk}\overline{b_k b_{j,p}}$ in the anisotropic case. (Here, $\boldsymbol{b} = \boldsymbol{B} - \overline{\boldsymbol{B}}$ is the deviation from the mean magnetic field, i.e. the fluctuating field, and $\boldsymbol{j} = \boldsymbol{\nabla} \times \boldsymbol{b}$ is the fluctuating current density, where the vacuum permeability is put to unity.)

The reason this term has not been included in the past is that it does not normally occur in the standard first order smoothing approximation that has frequently been used for calculating the α effect. However, when the so-called minimal tau approximation is used (Blackman & Field 2002; Rädler et al. 2003; see review by Brandenburg & Subramanian 2005a) this term appears quite naturally. Kleeorin & Rogachevskii (1999) have already used the τ approximation much earlier, and the $\overline{\boldsymbol{j} \cdot \boldsymbol{b}}$ correction was also used by Gruzinov & Diamond (1994) and Bhattacharjee & Yuan (1995), referring to original work of Pouquet et al. (1976), who were the first to use Orszag's (1970) τ approximation in MHD. Simulations have now verified explicitly the existence of the $\overline{\boldsymbol{j} \cdot \boldsymbol{b}}$ term (Brandenburg & Subramanian 2005b).

Once the proper course of the catastrophic quenching phenomenon was discovered, it became relatively easy to identify possible remedies, such as the allowance for helicity fluxes. However, it is not enough to allow for open boundaries; e.g. in a box with open boundaries such quenching yields saturation field strengths that depend on the mag-

netic Reynolds number (Brandenburg & Dobler 2001). It is necessary to have, throughout the domain, an active driver of helicity flux (magnetic or current helicity), for example shear (Vishniac & Cho 2001; Subramanian & Brandenburg 2004, 2006), or simple advection (Shukurov et al. 2006). In Fig. 5 we demonstrate the importance of open versus closed boundaries in a simulation of forced turbulence with shear (Brandenburg 2005). The simulation shown here has a shear profile that is relevant to a local model of part of the solar convection zone, but it is expected that the same physics carries over to large-scale dynamo action in accretion discs.

4 Conclusions

Not all magnetic fields will be helical, but if they are, this can have dramatic consequences for their evolution. The effects can be equally dramatic both in decaying and in driven turbulence, as has been highlighted in this review. Although we have not discussed this in the present paper, it should be emphasized that helical large-scale magnetic fields can also be generated in non-stratified shear flows where there is no α effect, but there can instead be the so-called shear–current of $\overline{W} \times \overline{J}$ effect (Rogachevskii & Kleeorin 2003, 2004). This effect may also explain the large-scale dynamo action seen in Fig. 5, where the results without helicity are quite similar to those with helicity (Brandenburg 2005). One-dimensional mean field calculations with the $\overline{W} \times \overline{J}$ effect (Brandenburg & Subramanian 2005c) show that in this case a magnetic α effect can be produced that has different signs on the two sides of the midplane. This magnetic α effect thus contributes to the saturation of the dynamo even if there is no ordinary (kinetic) α effect. This highlights once more the dramatic effects played by magnetic helicity.

Whether or not the primordial magnetic field was really helical remains a big question. If it was, it is likely that an inverse cascade process has produced fields of progressively larger scale. This might lead to observable effects in the cosmic microwave background. Such a field may also be important for seeding the galactic dynamo, but it is important to realize that a variety of astrophysical mechanisms may also produce seed fields just as large. Our estimate for magnetized outflows from AGNs or YSOs assumes that the source remains active for a certain period of time, and that their exhaust goes freely into the ambient medium. Partial evidence for this actually happening lies in the fact that clusters of galaxies are chemically enriched with heavier elements. Given that magnetic fields are intrinsically connected with the outflow, just like the heavier elements in it, it is quite plausible that some degree of magnetic contamination of the cluster must have occurred.

In order to produce finally the observed large-scale magnetic fields of galaxies, some more reshaping, amplification, and maintenance against magnetic decay is necessary. Roughly, we expect this to happen just like the mean field dynamo is able to amplify and maintain the field, although it must operate on an already strong enough field. This ini-

tial field will still be random and of mixed parity about the midplane (or equator), but there will be some finite degree of quadrupolar field which is the one that is dominant in many galaxies; see Krause & Beck (1998) and Brandenburg & Urpin (1998) for a related discussion about the importance of seeding the quadrupolar field component. As we have argued above, the catastrophic quenching problem of the dynamo has to be overcome, and this is likely to be the case because of various magnetic and current helicity fluxes operating within the entire dynamo domain. In the context of the solar dynamo, simulations have now begun to demonstrate the dramatic difference made by open boundary conditions, and we hope that a similar demonstration will soon be possible for the galactic dynamo as well. Corresponding mean field calculations have already been performed showing that the catastrophic quenching effect is overcome by an advective flux out of the domain along the vertical direction. In particular, it will be interesting to see whether the shedding of magnetic helicity can actually lead to directly observable effects.

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