Schwinger effect in axion inflation on a lattice

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We present the first lattice simulations of the nonlinear evolution after axion inflation by selfconsistently incorporating currents arising from Schwinger pair production. The tachyonically amplified gauge fields trigger the growth of Schwinger currents, leading to universal values for the conductivity and magnetic field at the onset of strong backreaction and subsequent quenching of gauge field production. We show that the Schwinger effect (prematurely) saturates gauge field production, thereby diminishing the prospects of axion inflation magnetogenesis as a viable solution for blazar observations.

Introduction. The inflationary paradigm provides a compelling framework for understanding the origin of large-scale structure and the observed homogeneity and isotropy of the universe [1–3]. Despite the abundance of inflationary models, axion (or natural) inflation [4, 5] has been receiving attention due to its theoretical robustness and its natural embedding within string theory and other ultraviolet (UV) complete frameworks [6–11]. In such models, the inflaton is identified with a pseudo-scalar axion-like field, which naturally enjoys a shift symmetry—crucial for maintaining a flat potential across super-Planckian field excursions.

The shift symmetry of the axion requires it to couple only derivatively to gauge fields or fermions. Chern-Simons couplings $\phi F\tilde{F}$ lead to the exponential amplification of gauge field modes during and after inflation [12, 13]. This amplification can generate distinctive non-Gaussian signatures [13–15], chiral gravitational waves [16–22], primordial magnetic fields [23–27] and lead to almost instantaneous preheating [28–30]. Recent lattice simulations have been performed to revisit the rich phenomenology emerging from axion-gauge field interactions [30–38]. Furthermore, the induced electric fields from the amplified gauge modes can become large enough to trigger non-perturbative pair production of charged particles via the Schwinger effect [39–42]. Due to its non-perturbative and nonlinear nature and its importance for axion inflation, capturing the dynamics of the Schwinger effect has attracted significant attention and several methods have been proposed [43–49].

This Letter contains results from the first lattice simulation of preheating after axion inflation, where the Schwinger effect is self-consistently taken into account. We demonstrate a suppression of the produced electric and magnetic fields, effectively ruling out primordial magnetogenesis from axion inflation. We discover a universal value for both the electromagnetic (EM) fields as well as the conductivity of the Schwinger plasma at the onset of backreaction and present a simple derivation of these values, based on the competition between the axion-gauge coupling term and the Schwinger current in the equation of motion of the electric field. Furthermore, we explore the effect of the mass of the lightest Standard Model fermions and show that a large Higgs vacuum expectation value (VEV) during inflation can restore the viability of axion inflation magnetogenesis.

The rest of the Letter is organized as follows. We start by presenting the basics of the model and the different descriptions of the Schwinger-induced current. Following that, we present the results of our numerical simulations and analytic estimates. We conclude with the limitations of our method and outlook for future work.

Axion inflation and the Schwinger effect. We consider a pseudoscalar inflaton (axion) ϕ coupled to the hypercharge sector of the Standard Model through a Chern-Simons interaction term in the presence of charged particles

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\alpha}{4f} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} + \mathcal{L}_{ch} \right], \qquad (1)$$

where $F_{\mu\nu} = \partial_{\mu}A^{\rm ph}_{\nu} - \partial_{\nu}A^{\rm ph}_{\mu}$, α is the axion-gauge cou-

pling, f is the axion decay constant, $V(\phi)$ is an axion potential and $\mathcal{L}_{ch} = \mathcal{L}_{ch}(A^{\rm ph}_{\mu}, \chi)$ describes all charged fields, χ , and their interaction with $A^{\rm ph}_{\mu}$. With the superscript "ph" we denote *physical* fields. The physical electric four-current is then $J^{\mu} = -\partial \mathcal{L}_{ch}/\partial A_{\mu} = (\rho_{ch}, J^{\rm ph}/a)$, where a(t) is the scale factor and we assume charged particles initially absent (or exponentially diluted during inflation) and thus set the initial charge density to zero, $\rho_{ch} = 0$. It is convenient to work with *comoving* fields that relate to physical as $E = a^2 E^{\rm ph}, B = a^2 B^{\rm ph},$ $J = a^3 J^{\rm ph}$. Comoving electric and magnetic fields are defined as $E = -\partial_{\tau} A + \nabla A_0, B = \nabla \times A$, where we use derivatives with respect to conformal time $d\tau = dt/a(t)$. The dynamical equations that govern the evolution of the (comoving) gauge and axion fields are [43, 45, 46]

$$\partial_{\tau}^{2}\phi + 2\mathcal{H}\partial_{\tau}\phi - \boldsymbol{\nabla}^{2}\phi + a^{2}\frac{dV}{d\phi} = \frac{\alpha}{a^{2}f}\boldsymbol{E}\cdot\boldsymbol{B},$$
 (2)

$$\partial_{\tau} \boldsymbol{E} - \operatorname{rot} \boldsymbol{B} + \frac{\alpha}{f} \left(\partial_{\tau} \phi \boldsymbol{B} + \boldsymbol{\nabla} \phi \times \boldsymbol{E} \right) + \boldsymbol{J} = 0, \quad (3)$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = -\frac{\alpha}{f} \boldsymbol{\nabla} \phi \cdot \boldsymbol{B}, \quad \boldsymbol{\nabla} \cdot \boldsymbol{B} = 0, \tag{4}$$

$$\partial_{\tau} \boldsymbol{B} + \operatorname{rot} \boldsymbol{E} = 0, \tag{5}$$

$$\mathcal{H}^2 = \frac{8\pi}{3m_{\rm Pl}^2} a^2 \left(\rho_\phi + \rho_E + \rho_B + \rho_\chi\right),\tag{6}$$

where $\mathcal{H} = \partial_{\tau} a/a$ is the conformal Hubble parameter. The energy densities are defined as $\rho_{\phi} = \langle (\partial_{\tau}\phi)^2/2a^2 + (\nabla\phi)^2/2a^2 + V \rangle$ for the axion, $\rho_E = \langle \mathbf{E}^2 \rangle/2a^4$ for the electric field, $\rho_B = \langle \mathbf{B}^2 \rangle/2a^4$ for the magnetic field, and ρ_{χ} for the the plasma. In the simulation, $\langle \dots \rangle$ denotes volume averaging over the whole simulation domain.

Strong backreaction from Schwinger currents. The induced Schwinger current generated by the created particles for the case of constant and spatially uniform (anti)collinear electric and magnetic fields in de Sitter space takes the form [43, 45, 49, 50]

$$J = \frac{(e|Q|)^3}{6\pi^2 \mathcal{H}} E|B| \coth\left(\frac{\pi|B|}{E}\right) e^{-\frac{\pi m^2 a^2}{e|Q|E}},\tag{7}$$

where $E = |\mathbf{E}|$ is the magnitude of the electric field and J, B are the electric current and magnetic field, projected onto the direction of the electric field, e is the gauge coupling constant, Q is the particle's charge, and m is the particle's mass. We focus our attention on the strong-field limit, defined as [49] $|eQE| \gg \mathcal{H}^2$, meaning that we choose couplings that would generate an E-field that satisfies the above inequality in the absence of a Schwinger plasma. We also neglect the fermion masses by assuming $m\pi a^2 \ll e|Q|E$, unless otherwise stated. When electric

and magnetic fields are (anti-)collinear, the induced current is proportional to both E and B. This results in an ambiguity in writing a vector form for the Ohm's law for the Schwinger current and allows for different formulations, dubbed the "electric", "magnetic" and "mixed" picture

$$\boldsymbol{J} = \sigma_E \boldsymbol{E}, \quad \sigma_E = \frac{(e|Q|)^3}{6\pi^2 \mathcal{H}} |B| \coth\left(\frac{\pi B}{E}\right), \tag{8}$$
$$\boldsymbol{J} = \sigma_B \boldsymbol{B}, \quad \sigma_B = \frac{(e|Q|)^3}{6\pi^2 \mathcal{H}} \operatorname{sign}(\boldsymbol{E} \cdot \boldsymbol{B}) E \coth\left(\frac{\pi B}{E}\right), \tag{9}$$

$$\boldsymbol{J} = \sigma_E \boldsymbol{E} + \sigma_B \boldsymbol{B},\tag{10}$$

where for the mixed picture the conductivities σ_E , σ_B are chosen to satisfy Eq. (7). We refer to this description of conductivities as *collinear*, to emphasize the underlying assumption of (anti-)collinearity of the fields.

However, in axion inflation, electric and magnetic fields may not remain collinear or anti-collinear at all times. Relaxing the assumption of collinearity was addressed by performing a Lorentz boost from the comoving coordinate frame to a frame in which the electric and magnetic fields are collinear, and then transforming back. This was first explored perturbatively by considering small deviations around constant, anti-collinear background fields in Ref. [47], and later extended to a non-perturbative treatment in Ref. [49]. Since no assumption is made about the collinearity of the fields, and they can take arbitrary configurations, we refer to this description as *non-collinear*. This procedure leads to the induced current in the mixed picture (10), where it is described through both an electric and magnetic conductivities as [49]

$$\sigma_E = \frac{|\boldsymbol{J}'|E'}{\gamma I^2}, \quad \sigma_B = \frac{|\boldsymbol{J}'|}{E'\gamma I^2} (\boldsymbol{E} \cdot \boldsymbol{B}), \qquad (11)$$

where $I^2 \equiv \sqrt{(E^2 - B^2)^2 + 4(E \cdot B)^2}$ and prime quantities are fields in the collinear frame defined through an arbitrary configuration of comoving E and B fields as

$$J' = \frac{(e|Q|)^3}{6\pi^2 \mathcal{H}} E'|B'| \coth\left(\frac{\pi|B'|}{E'}\right), \qquad (12)$$

$$E' = \frac{1}{\sqrt{2}} \left[\boldsymbol{E}^2 - \boldsymbol{B}^2 + I^2 \right]^{1/2}, \qquad (13)$$

$$B' = \frac{\operatorname{sign}(\boldsymbol{E} \cdot \boldsymbol{B})}{\sqrt{2}} \left[\boldsymbol{B}^2 - \boldsymbol{E}^2 + \boldsymbol{I}^2 \right]^{1/2}, \qquad (14)$$

$$y = \frac{1}{\sqrt{2}} \left[1 + \frac{E^2 + B^2}{I^2} \right]^{1/2} .$$
 (15)

To obtain a closed system of equations, one needs to account for the evolution of the fermion energy density, ρ_{χ} . Incorporating energy conservation in an expanding universe, the equation for ρ_{χ} can be written phenomenologically as [44, 49]

$$\partial_{\tau}\rho_{\chi} + 4\mathcal{H}\rho_{\chi} = \frac{1}{a^3} \left(\langle \sigma_E \rangle \langle \boldsymbol{E}^2 \rangle + \langle \sigma_B \rangle \langle \boldsymbol{E} \cdot \boldsymbol{B} \rangle \right), \quad (16)$$

where it is assumed that the plasma is comprised of relativistic particles possessing a statistically isotropic momentum distribution, $p_{\chi} = \rho_{\chi}/3$. It is worth noting that in Eq. (16) one could use " $\langle \sigma_E \mathbf{E}^2 \rangle + \langle \sigma_B \mathbf{E} \cdot \mathbf{B} \rangle$ ". We defer a detailed comparison of different prescriptions for a subsequent publication.

Numerical simulations. To determine the evolution of the system, we solve Eqs. (2)–(6) together with Eq. (16). This is done numerically on a lattice using the PEN-CIL CODE [51]. For our choice $\alpha m_{\rm Pl}/f = 60$, a grid of 512³ points is sufficient. We start around 3 *e*-folds before the end of inflation and initialize fields with the Bunch-Davies initial conditions. For simplicity we choose a quadratic potential for the axion $V(\phi) = \frac{1}{2}m^2\phi^2$ with $m = 1.04 \times 10^{-6} m_{\rm Pl}$. Even though this is observationally ruled out during inflation, it is a valid approximation during preheating and as such has been widely used in the literature [35–38]. We do not expect qualitative differences for more complicated potentials, like axion monodromy [28].

As outlined above, there are several possible descriptions of the Schwinger current. This is due to the nonperturbative nature of the effect and the fact that the solution is only known for a constant electric field limit. Thus extrapolating from this to more realistic scenarios leads to different prescriptions. We begin our analysis with the simplest parametrization: the collinear current description in the electric picture, given by Eq. (8), and compare it with the non-collinear formulation Eq. (10) – (11). We follow the definitions of Ref. [49] for the charge and the gauge coupling constant. In the expression for the conductivities we set $Q^3 = 41/12$, which equals half the sum of the cubes of the hypercharges of all Standard Model Weyl fermions (while Eq. (7) refers to a single Dirac fermion). The gauge coupling constant is $e = q' = \sqrt{4\pi/137} \simeq 0.303$, but a realistic description of the Schwinger effect requires taking into account its running. Hence in our simulations we use the gauge coupling constant $e = q'(\tilde{\mu})$ defined as

$$g'(\tilde{\mu}) = \left([g'(m_Z)]^{-2} + \frac{41}{48\pi^2} \ln \frac{m_Z}{\tilde{\mu}} \right)^{-1/2}, \qquad (17)$$

where $g'(m_Z) \simeq 0.35$, $m_Z \simeq 91.2 \,\text{GeV}$, with the characteristic energy scale $\tilde{\mu}$

$$\tilde{\mu} = (\rho_E + \rho_B)^{1/4} = \frac{1}{a} \left(\frac{1}{2} \langle \boldsymbol{E}^2 \rangle + \frac{1}{2} \langle \boldsymbol{B}^2 \rangle \right)^{1/4}.$$
 (18)

The conductivities in Eqs. (8) and (11) depend on the electric and magnetic fields. In our numerical simulations, the conductivity is computed *locally* from the fields at each grid point. Nevertheless, defining it in terms of spatially averaged fields yields nearly identical results (detailed comparison will be presented in a subsequent publication).

We perform fully nonlinear simulations for the collinear (Eq. (8)) and non-collinear (Eq. (10) - (11)) description of conductivities, consistently applying the local formulation in the equations of motion. For comparison, we also consider the evolution in the strong backreaction regime without fermions (see e.g. [26, 37]). Furthermore, we provide a comparison to the linear approximation with homogeneous inflaton dynamics, where $\boldsymbol{E} \cdot \boldsymbol{B}$ and $\nabla \phi$ are omitted in Eq. (2). The Hubble scale in the last case is taken to depend only on the inflaton $\mathcal{H}^2 = (8\pi/3m_{\rm Pl}^2) a^2 \rho_{\phi}$. The result of our simulations is shown in Figure 1. On the top panel we show the evolution of the root-mean-square (rms) magnetic field strength, defined as $B_{\rm rms} = \sqrt{\int d\log k \cdot P_B(k)}$, where $P_B(k) \equiv 4\pi k^2 |B(k)|^2$. We also show the evolution of the rms electric field, $E_{\rm rms}$, defined analogously. We provide this comparison for collinear and non-collinear cases.

We see that all cases which operate in the largecoupling regime exhibit very similar suppression, regardless of the current description or whether the regime is treated as linear or nonlinear. This indicates that the presence of the Schwinger current prevents the gauge field from entering the strong backreaction regime, leading to a universal suppression. Consequently, Schwinger-induced damping can be estimated within the linear approximation, allowing for the use of semi-analytical methods.

On the lower panel of Fig. 1 we show the evolution of conductivities. We see that taking the collinear prescription, the electric conductivity is the same for the linear and non-linear cases, whereas in the non-collinear prescription the electric and magnetic conductivities are oscillating in time.

Universality of the Schwinger backreaction. It is known that the largest amplification of gauge fields during axion inflation occurs close to the end of inflation [28]. Therefore, as the tachyonic amplification depends on the axion velocity, which is maximal close to the end of inflation. The growth of **E** and **B** can be described by a simple exponential growth rate (see [28] for a WKB analysis). Furthermore, both fields are almost equal during this growth. By examining the equation of motion for the gauge field (Eq. 3) we see two competing terms: $(\alpha/f)(\partial_{\tau}\phi)\mathbf{B}$ supports the tachyonic amplification, whereas the current $\mathbf{J} = \sigma_E \mathbf{E}$ opposes it. Initially



FIG. 1. Upper panel: The rms values of the electric and magnetic fields under different assumptions. Magnetic fields are shown for the following cases: linear approximation with a homogeneous inflaton and no Schwinger current (red solid), nonlinear backreaction with a nonhomogeneous inflaton and no Schwinger current (black solid), collinear conductivity description in the linear (orange) and nonlinear (black dashed) regimes, and non-collinear conductivity (blue solid). Finally, the brown-dashed and green-solid curves are the rms electric fields that correspond to the black-dashed and blue-solid $B_{\rm rms}$ field results (collinear and non-collinear prescriptions, respectively). Lower panel: Electric conductivity in the collinear description is shown for the linear case (red solid) and the nonlinear backreaction case (black dashed). The blue and green solid lines correspond to the non-collinear conductivity prescription. Color coding matches the upper panel.

the tachyonic amplification term dominates and thus one of the two polarizations undergoes the usual exponential enhancement. Since $E \approx B$, we can compare the two terms by comparing $(\alpha/f)\partial_{\tau}\phi$ to the conductivity σ_E . Using $\partial_{\tau}\phi = m_{\rm Pl}\mathcal{H}\sqrt{\epsilon/4\pi}$ we get $(\alpha/f)\partial_{\tau}\phi \sim \mathcal{O}(100)\mathcal{H}$, where we took $\epsilon \sim 1$ close to the end of inflation and $\alpha m_{\rm Pl}/f \sim 60 - 100$. For $\mathcal{H} \sim 10^{-5} m_{\rm Pl}$ we see that the backreaction from Schwinger pair production occurs at $\sigma_E \sim 10^{-3} m_{\rm Pl}$. We can now also estimate the typical value of the electric and magnetic fields. First we observe that $\coth(\pi E/B) \simeq 1$ for $E/B = \mathcal{O}(1)$. Furthermore $eQ \simeq 1$, leading to $B \sim 6\pi^2 (\alpha m_{\rm Pl}/f) \mathcal{H}^2 \sim 10^{-6} m_{\rm Pl}^2$.

Intriguingly, the above estimates for the conductivity and the value of the EM fields at the onset of Schwinger backreaction are consistently supported by a wide range of simulations. In the large coupling regime the universe preheats almost instantaneously such that the inflaton transfers all its energy to gauge fields. In the absence of the Schwinger effect, this universal plasma conductivity is reached during the early gauge field growth at the end of inflation. This is true for the large amplitude cases $\alpha m_{\rm Pl}/f = 60,75,90$ [52]. Our analysis points to the existence of a universal behavior for axion inflation magnetogenesis, where the Schwinger effect is significant when gauge fields reach a value of $E, B \sim \mathcal{O}(10^{-6}) m_{\rm Pl}$ close to the end of inflation. The Schwinger suppression will be less pronounced (largely irrelevant) for couplings that lead to weaker fields.

Consequences for magnetogenesis. The nondetection of secondary GeV photons from blazars provides indirect evidence for the presence of extragalactic magnetic fields in the intergalactic medium (possibly helical [53]). This observation motivates investigations into their origin in the early universe. The prospect of magnetogenesis in axion inflation has been explored for couplings up to $\alpha m_{\rm Pl}/f \leq 60$ in Ref. [26] and, more recently, for $\alpha m_{\rm Pl}/f = 75$, 90 in Ref. [37]. These studies conclude that, for $\alpha m_{\rm Pl}/f \ge 60$, the axion–U(1) inflation model is already marginally compatible with generating magnetic fields strong enough to account for the non-observation of GeV photons in blazar spectra. Moreover, as we have seen, the Schwinger effect significantly reduces the final amplitude. To quantify the suppression, let us consider the present-day magnetic field strength and its coherence length. After accounting for the nonlinear evolution of the fields (including inverse cascade [54, 55]) they are given by [37]

$$B_{\rm rms}^{\rm ph}|_{0} = 9.2 \times 10^{-15} \,\mathrm{G}\sqrt{\frac{\int d\log k \cdot P_{B}}{\rho_{\rm tot}}} \left(\frac{10^{-6}m_{\rm Pl}}{H}\right) r_{A}^{1/3}$$
(19)

$$L_c|_0 = 0.8 \,\mathrm{p}c \left(\mathcal{H}L_c\right) \left(\frac{10^{-6} m_{\mathrm{P}l}}{H}\right) r_A^{-2/3},\tag{20}$$

where H is the Hubble parameter in cosmic time, $\rho_{\text{tot}} = \rho_{\phi} + \rho_{\text{E}B} + \rho_{\chi}$ is the total energy density at the end of the simulation, $L_c = \frac{\int d \log k \cdot P_B / k}{\int d \log k \cdot P_B}$ is the coherence length, and the parameter $r_A = \max(1, \mathcal{H}L_c/V_A)$,



FIG. 2. The ratio of the EM energy density $\rho_{\rm EB} \equiv \rho_E + \rho_B$ and the plasma energy density ρ_{χ} to the total energy density $\rho_{\rm tot}$ in the simulation box for runs with $\alpha m_{\rm Pl}/f = 60$. The black solid curve corresponds to a case without the Schwinger effect and the black-dashed curve to a case with the Schwinger effect and massless fermions. The corresponding plasma energy density ratio is shown in red-solid. The EM energy density ratio for fermion masses $m = 10^{-4}, 10^{-2} m_{\rm Pl}$ are shown in solid orange and blue, respectively.

where $V_A = \sqrt{B^2/(\rho_{\rm tot} + p)}$ is the Alfvén velocity. The lower bound on the present-day magnetic field strength depends on the coherence length as $B_{\text{bound}} \approx 2 \times 10^{-17} \,\text{G} \left(L_c|_0/0.2 \,\text{Mpc}\right)^{1/2}$ [56]. The results of [37] show that even without the Schwinger suppression for $\alpha m_{\rm Pl}/f \geq 60$ and a coherence length $L_c|_0 \simeq 10^{-1}$ 10^{-2} pc, the primordial magnetic field has an amplitude $B_{\rm rms}^{\rm ph}|_0 \simeq 10^{-14}$ -10⁻¹⁵G, which is marginally consistent with the observational lower bound of $B_{\text{bound}} \simeq 10^{-14} \,\text{G}$ for this coherence length. When the Schwinger effect is taken into account, it suppresses $B_{\rm rms}$ by at least two orders of magnitude for large couplings, and reduces the EM energy density by approximately four orders of magnitude. As a result, the ratio $B_{\rm rms}/\sqrt{\rho_{\rm tot}}$ in (19) remains well below unity, effectively ruling out magnetogenesis from axion inflation. This is shown in Figure 2 for $\alpha/f = 60m_{\rm P}$. We see that even though in the absence of the Schwinger effect the entirety of the energy density ends up in EM fields, once the Schwinger effect is considered the relativistic plasma takes a maximum of $\mathcal{O}(10\%)$ of the energy density with the EM fields being much more suppressed. Therefore the inflaton keeps dominating (until it perturbatively decays to photons). The produced B field reaches a higher value if a different prescription of the Schwinger current is employed, where the differential equation for $\partial_{\tau} J$ is solved numerically instead of relying on Eq. (7) [57]

Heavy fermion effects. So far, we have neglected the effects arising from finite fermion masses. From (7) it follows that a significant suppression of the Schwinger effect requires $\pi m^2 a^2 > e |Q| E$ or $m^2/E_{\rm ph} \gtrsim \mathcal{O}(1)$, where $E_{\rm ph}$ is the amplitude of the physical electric field. We are focusing on the era close to the end of inflation, where $B_{\rm ph}^2 \simeq E_{\rm ph}^2 \sim 3H^2 m_{\rm Pl}^2/8\pi$, or $E_{\rm ph} \sim 0.1 H m_{\rm Pl}$. The masses of Standard Model (SM) fermions are given by m = yh, where y is the Yukawa coupling and h is the Higgs VEV, which is expected to be nonzero during inflation, if the Higgs is a light field subject to de-Sitter fluctuations. Among the electrically charged fermions, the electron has the smallest Yukawa coupling, $y_e \simeq 3 \times 10^{-6}$, making it the lightest. Thus, if electrons are too heavy to be efficiently produced via the Schwinger effect, all other charged fermions will be even more suppressed. To suppress the Schwinger effect, the fermion mass must satisfy

$$m^2 \gtrsim 0.1 H m_{\rm Pl} \,. \tag{21}$$

Figure 2 shows how the presence of a large electron mass suppresses the Schwinger current and allows for the generation of a larger magnetic field. However, the constraint (21) implies a lower bound on the Higgs VEV $h \gtrsim (0.3/y_e)\sqrt{Hm_{\rm Pl}} \simeq 10^5 m_{\rm Pl}\sqrt{H/m_{\rm Pl}}$. To avoid super-Planckian values for the Higgs field, one requires a low inflationary scale: $H \lesssim 10^{-10} m_{\rm Pl} \sim 10^9 \,{\rm GeV}$. This provides a key result: a suppression of the Schwinger effect through fermion masses requires both low-scale inflation and large (possibly Planckian) field excursions for the Higgs. If one relies solely on de Sitter fluctuations to generate a Higgs VEV, one expects $h \sim H\lambda^{-1/4}$, which leads to $H/m_{\rm Pl} \gtrsim 10^{12} \sqrt{\lambda}$. This requirement is difficult to satis fy given observational upper bounds on H, unless we consider an almost vanishing Higgs self-coupling. However, alternative mechanisms such as a direct coupling between the Higgs and the inflaton, or a non-minimal coupling to gravity can dynamically induce a large Higgs VEV during inflation.

These considerations point to an intriguing modelbuilding challenge: any realistic suppression of the Schwinger effect involving SM fermions may lead to observable consequences at collider experiments through a modification of the Higgs sector. Before concluding, we must note that all simulations presented here refer to high-scale inflation, $H \sim 10^{-6} m_{\rm Pl}$, and thus need to be re-done for different Hubble scales. In particular, Figure 2 shows that the Schwinger effect is still active if we consider large fermion masses, albeit weaker. That being said, reducing the scale of inflaton allows for a large hierarchy between the Hubble scale and thus the possible electric field values are reduced, whereas the fermions can be equally heavy, if we take the Higgs VEV close to the Planck mass during inflation. Therefore, a definitive conclusion on the viability of axion inflation magnetogenesis requires a dedicated parameter scan, which we defer for future work.

Summary and outlook. We presented the first lattice simulations of the nonlinear evolution after axion inflation that self-consistently incorporate Schwinger pair production. Our results demonstrate that the induced Schwinger current provides a robust backreaction that quenches gauge field amplification once a universal critical conductivity $\sigma_E \sim 10^{-3}m_{\rm Pl}$ and magnetic field strength $B_{\rm rms} \sim 10^{-6}m_{\rm Pl}^2$ are reached. These critical values are independent of the specific formulation of the current and mark the onset of plasma domination.

The resulting suppression of EM field production significantly reduces the amplitude of primordial magnetic fields, effectively ruling out axion inflation as a source of intergalactic magnetogenesis. Avoiding strong suppression requires fermion masses large enough to inhibit Schwinger production—necessitating a large Higgs VEV during inflation and favoring low-scale inflation scenarios. These findings motivate future studies on low-scale inflationary models and on connections between the axion and Higgs sectors, with potential implications for collider signatures. Finally, a different formulation of the Schwinger current, based on a numerical integration of $\partial_{\tau}J$ provides an increased *B* field, warranting further investigation.

Data availability. The source code used for the simulations of this study, the PENCIL CODE, is freely available from Refs. [51, 58]. The simulation setups and the corresponding data are freely available from Ref. [59].

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the B field for $\alpha m_{\rm P}/f = 60,90$ differs by less than an order of magnitude, making our threshold estimates robust.

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Schwinger effect in axion inflation on a lattice

Supplemental Material

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Evolution with different couplings

In this section we demonstrate the evolution of the magnetic field and electric conductivities for $\alpha m_{\rm Pl}/f = 90,75,60$. For simplicity we chose the collinear case for conductivities given by Eq. (8). As illustrated in Figure S1, the impact of Schwinger suppression remains within an order-of-magnitude variation at strong coupling.



FIG. S1. Comparison of $B_{\rm rms}$ and σ_E for cases with $\alpha m_{\rm Pl}/f = 90,75,60$ (solid orange, solid blue, dashed black curves) for the collinear case and full nonlinear evolution.

Full integration of the current

Let us start by providing a heuristic derivation of the Schwinger current [41, 62] in Minkowski space. We first compute the pair production rate [63] (for weak fields) $\Gamma \propto (eE)^2 e^{\frac{-\pi m^2}{eE}}$. These particles are accelerated by the electric field through $v = \frac{e}{m} \int E \, dt$. Combining these leads to an equation for the current as an integral over time, or equivalently a first-order differential equation where dJ/dt is a function of E. Performing a more careful calculation in an expanding space-time similarly leads to the first-order differential equation for the dynamical current [43]

$$\partial_{\tau}J = \frac{(e|Q|)^3}{2\pi^2} E|B| \coth\left(\frac{\pi|B|}{E}\right), \qquad (S1)$$

where the fermion mass is dropped for simplicity and the fields are comoving as in the main text. One can integrate the above, by assuming constant (physical) EM fields and constant Hubble scale, and arrive at

$$J = \frac{(e|Q|)^3}{6\pi^2 \mathcal{H}} E|B| \coth\left(\frac{\pi|B|}{E}\right)$$
(S2)

which is the form of the current that we used in our simulations and has been widely used in the literature [43–45, 47, 49].

Since we are evolving the full system on the lattice, there is no added complication in numerically integrating Eq. (S1) instead of using Eq. (S2). Figure S2 shows the results of a simulation run, where the axion and EM fields



FIG. S2. Comparison of $E_{\rm rms}$, $B_{\rm rms}$, $J_{\rm rms}$, σ_E and $\rho_{EB}/\rho_{\rm tot}$ for cases with dynamical (solid green) and non-dynamical (dashed black) current. The axion-gauge coupling is chosen as $\alpha m_{\rm Pl}/f = 60$. In the upper left panel, solid (dashed) curves correspond to B(E) fields. The solid black curve on the lower left panel corresponds to the EM energy density case without Schwinger suppression.

are computed at each point on the lattice, while the current is computed by integrating Eq. (S2), similarly at each point on the lattice. We first of all observe that until the universal threshold of $(\sigma_E, B) \sim (10^{-3}, 10^{-6} m_{\rm Pl}^2)$, the evolution of the system using either prescription is very similar. The conductivity σ_E continues to be very similar in the two cases. However, the electric and magnetic fields show significant differences. After they reach their peak at $E_{\rm rms}, B_{\rm rms} = \mathcal{O}(10^{-6}) m_{\rm Pl}$, their behavior is qualitatively different in the two cases. Using the prescription studied in the main part of the paper, Eq. (S2), the electric and magnetic fields decay (redshift) after reaching their threshold value. However, when we instead integrate numerically Eq. (S1), we see that the EM fields keep growing past their threshold values. This implies that they continue to extract energy from the plasma itself. It is important to recall that the time derivative of the EM energy is equal to $-\langle \mathbf{J} \cdot \mathbf{E} \rangle$. With the usual Ohm's law, $\mathbf{J} = \sigma_E \mathbf{E}$, we find that $\langle \mathbf{J} \cdot \mathbf{E} \rangle = \langle \mathbf{J}^2 \rangle / \sigma_E$ is positive definite, so EM energy is always being dissipated into heat and thus contributes to reheating the universe. Even if the result of integrating Eq. (S1) seems counter-intuitive, since the plasma is expected to quench the growth of EM fields, not fuel it, the prescription used can actually support it.

Using (S1) we have $\langle \mathbf{J} \cdot \mathbf{E} \rangle = \partial_{\tau} \langle \mathbf{J}^2 \rangle / 2\tilde{\sigma}_E$, where $\tilde{\sigma}_E$ denotes the right-hand side of (S1). Thus, $\langle \mathbf{J} \cdot \mathbf{E} \rangle$ is no longer positive definite, but, since we find that $J_{\rm rms}$ still increases during the entire evolution seen in Fig. S2, there is still positive energy dissipation. Put in a more pictorial way, when integrating the current and using Eq. (S2), the direction of the current is always opposing the electric fields. This is also easily understood in the case of static fields, for which this whole formalism was developed. In the case of time-varying fields though, using Eq. (S1), means that the current evolution has its own time-scale and thus a "memory effect" can arise. Simply put, the direction of the current is co-decided by the electric field at each instance in time, but also by the history of the electric field.

At the end of our simulation $J_{\rm rms}$ is about two orders of magnitude larger than before, and yet, $E_{\rm rms}$ is not suppressed. Thus, the common expectation of conductivity short-circuiting the electric field [60] is not obeyed. For completeness, we also note that the Schwinger pair plasma will not remain at rest, but it can be accelerated by the Lorentz force, $\mathbf{J} \times \mathbf{B}$. This leads to some of the EM energy being converted into kinetic energy at the rate $\langle \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) \rangle$ where \mathbf{u} is the bulk velocity of the plasma [61]. Addressing this is an important task for future studies.