

Schwinger effect in axion inflation on a lattice

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We present the first lattice simulations of the nonlinear evolution after axion inflation by self-consistently incorporating currents arising from Schwinger pair production. The tachyonically amplified gauge fields trigger the growth of Schwinger currents, leading to universal values for the conductivity and magnetic field at the onset of strong backreaction and subsequent quenching of gauge-field production. We show that the Schwinger effect (prematurely) saturates gauge-field production, thereby diminishing the prospects of high-scale axion inflation magnetogenesis as a viable solution for blazar observations.

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I. INTRODUCTION

Inflation provides a compelling explanation for the origin of large-scale structure and the Universe's homogeneity and isotropy [1–3]. Among various models, axion (or natural) inflation [4,5] is particularly appealing due to its theoretical robustness and natural embedding in UV-complete frameworks like string theory [6–11]. The inflaton in these models is a pseudoscalar axionlike field, whose shift symmetry ensures a flat potential across super-Planckian field excursions. The shift symmetry restricts the axion to derivative couplings, such as Chern-Simons terms ($\phi F\tilde{F}$), which lead to exponential gauge-field amplification during and after inflation [12,13]. These amplified modes can generate distinctive non-Gaussianities [13–15], chiral gravitational waves [16–22], primordial magnetic fields [23–27], and nearly instantaneous preheating [28–30]. These phenomena have been revisited in recent lattice simulations [30–38].

Moreover, the induced electric fields can become strong enough to trigger Schwinger pair production [39–42]. Because of its nonlinear and nonperturbative nature, accurately capturing this effect in axion inflation has drawn considerable attention, motivating various proposals [43–49].

This paper contains results from the first lattice simulation of preheating after axion inflation that self-consistently includes the Schwinger effect. We demonstrate a suppression of the produced electric and magnetic fields, effectively ruling out primordial magnetogenesis from high-scale axion inflation [50]. We discover a universal value for both the electromagnetic (EM) fields as well as the conductivity of the Schwinger plasma at the onset of backreaction and present a simple derivation of these values, based on the competition between the axion-gauge coupling term and the Schwinger current in the equation of motion of the electric field. Furthermore, we explore the effect of the mass of the lightest Standard Model fermions and show that a large Higgs vacuum expectation value (VEV) during inflation can weaken the Schwinger backreaction, partially restoring gauge-field growth.

The rest of the paper is organized as follows. We start by presenting the basics of the model and the different prescriptions of the Schwinger-induced current. Following that, we present the results of our numerical simulations and analytic estimates. We conclude with the limitations of our method and outlook for future work.

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II. AXION INFLATION AND THE SCHWINGER EFFECT

We consider a pseudoscalar inflaton (axion) ϕ coupled to the hypercharge sector of the Standard Model through a Chern-Simons interaction term in the presence of charged particles

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\alpha}{4f} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} + \mathcal{L}_{\text{ch}} \right], \quad (1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu^{\text{ph}} - \partial_\nu A_\mu^{\text{ph}}$, α is the axion-gauge coupling, f is the axion decay constant, $V(\phi)$ is an axion potential, and $\mathcal{L}_{\text{ch}} = \mathcal{L}_{\text{ch}}(A_\mu^{\text{ph}}, \chi)$ describes all charged fields χ and their interaction with A_μ^{ph} . With the superscript “ph” we denote physical fields. The physical electric four-current is then $J^\mu = -\partial \mathcal{L}_{\text{ch}} / \partial A_\mu = (\rho_{\text{ch}}, \mathbf{J}^{\text{ph}}/a)$, where $a(t)$ is the scale factor and we assume charged particles initially absent (or exponentially diluted during inflation) and thus set the initial charge density to zero, $\rho_{\text{ch}} = 0$. It is convenient to work with comoving fields that relate to physical as $\mathbf{E} = a^2 \mathbf{E}^{\text{ph}}$, $\mathbf{B} = a^2 \mathbf{B}^{\text{ph}}$, $\mathbf{J} = a^3 \mathbf{J}^{\text{ph}}$. Comoving electric and magnetic fields are defined as $\mathbf{E} = -\partial_\tau \mathbf{A} + \nabla A_0$, $\mathbf{B} = \nabla \times \mathbf{A}$, where we use derivatives with respect to conformal time $d\tau = dt/a(t)$. The dynamical equations that govern the evolution of the (comoving) gauge and axion fields are [43,45,46]

$$\partial_\tau^2 \phi + 2\mathcal{H} \partial_\tau \phi - \nabla^2 \phi + a^2 \frac{dV}{d\phi} = \frac{\alpha}{a^2 f} \mathbf{E} \cdot \mathbf{B}, \quad (2)$$

$$\partial_\tau \mathbf{E} - \text{rot } \mathbf{B} + \frac{\alpha}{f} (\partial_\tau \phi \mathbf{B} + \nabla \phi \times \mathbf{E}) + \mathbf{J} = 0, \quad (3)$$

$$\nabla \cdot \mathbf{E} = -\frac{\alpha}{f} \nabla \phi \cdot \mathbf{B}, \quad \nabla \cdot \mathbf{B} = 0, \quad (4)$$

$$\partial_\tau \mathbf{B} + \text{rot } \mathbf{E} = 0, \quad (5)$$

$$\mathcal{H}^2 = \frac{8\pi}{3m_{\text{Pl}}^2} a^2 (\rho_\phi + \rho_E + \rho_B + \rho_\chi), \quad (6)$$

where $\mathcal{H} = \partial_\tau a/a$ is the conformal Hubble parameter. The physical energy densities are $\rho_\phi = \langle (\partial_\tau \phi)^2 / 2a^2 + (\nabla \phi)^2 / 2a^2 + V \rangle$ for the axion, $\rho_E = \langle \mathbf{E}^2 \rangle / 2a^4$ for the electric field, $\rho_B = \langle \mathbf{B}^2 \rangle / 2a^4$ for the magnetic field, and ρ_χ for the plasma. In the simulation, $\langle \dots \rangle$ denotes volume averaging over the whole simulation domain. We use the temporal gauge, $A_0 = 0$.

III. STRONG BACKREACTION FROM SCHWINGER CURRENTS

The induced Schwinger current generated by the created particles for the case of constant and spatially uniform (anti)collinear electric and magnetic fields in de Sitter space takes the form [43,45,49,51]

$$J = \frac{(e|Q|)^3}{6\pi^2 \mathcal{H}} E |B| \coth \left(\frac{\pi |B|}{E} \right) e^{-\frac{\pi m^2 a^2}{e|Q|E}}, \quad (7)$$

where $E = |\mathbf{E}|$ is the magnitude of the electric field and J , B are the electric current and magnetic field, projected onto the direction of the electric field, e is the gauge-coupling constant, Q is the particle's charge, and m is the particle's mass. We focus our attention on the strong-field limit, defined as [49] $|eQE| \gg \mathcal{H}^2$, meaning that we choose couplings that would generate an E field that satisfies the above inequality in the absence of a Schwinger plasma. We also neglect the fermion masses by assuming $m\pi a^2 \ll e|Q|E$, unless otherwise stated. When electric and magnetic fields are (anti)collinear, the induced current is proportional to both $\mathbf{E}$ and $\mathbf{B}$. This results in an ambiguity in writing a vector form for Ohm's law for the Schwinger current and allows for different formulations, dubbed the electric, magnetic, and mixed pictures,

$$\mathbf{J} = \sigma_E \mathbf{E}, \quad \sigma_E = \frac{(e|Q|)^3}{6\pi^2 \mathcal{H}} |B| \coth \left(\frac{\pi B}{E} \right), \quad (8)$$

$$\mathbf{J} = \sigma_B \mathbf{B}, \quad \sigma_B = \frac{(e|Q|)^3}{6\pi^2 \mathcal{H}} \text{sign}(\mathbf{E} \cdot \mathbf{B}) E \coth \left(\frac{\pi B}{E} \right), \quad (9)$$

$$\mathbf{J} = \sigma_E \mathbf{E} + \sigma_B \mathbf{B}, \quad (10)$$

respectively, where for the mixed picture the conductivities σ_E , σ_B are chosen to satisfy Eq. (7). We refer to this description of conductivities as collinear, to emphasize the underlying assumption of (anti)collinearity of the fields.

However, in axion inflation, electric and magnetic fields may not remain collinear or anticollinear at all times. Relaxing the assumption of collinearity was addressed by performing a Lorentz boost from the comoving coordinate frame to a frame in which the electric and magnetic fields are collinear, and then transforming back. This was first explored perturbatively by considering small deviations around constant, anticollinear background fields in Ref. [47] and later extended to a nonperturbative treatment in Ref. [49]. Since no assumption is made about the collinearity of the fields, and they can take arbitrary configurations, we refer to this description as noncollinear. This procedure leads to the induced current in the mixed picture (10), where it is described through both electric and magnetic conductivities as [49]

$$\sigma_E = \frac{|\mathbf{J}'|E'}{\gamma I^2}, \quad \sigma_B = \frac{|\mathbf{J}'|}{E'\gamma I^2}(\mathbf{E} \cdot \mathbf{B}), \quad (11)$$

where $I^2 \equiv \sqrt{(\mathbf{E}^2 - \mathbf{B}^2)^2 + 4(\mathbf{E} \cdot \mathbf{B})^2}$ and prime quantities are fields in the collinear frame defined through an arbitrary configuration of comoving $\mathbf{E}$ and $\mathbf{B}$ fields as

$$J' = \frac{(e|\mathcal{Q}|)^3}{6\pi^2\mathcal{H}} E'|B'| \coth\left(\frac{\pi|B'|}{E'}\right), \quad (12)$$

$$E' = \frac{1}{\sqrt{2}}[\mathbf{E}^2 - \mathbf{B}^2 + I^2]^{1/2}, \quad (13)$$

$$B' = \frac{\text{sign}(\mathbf{E} \cdot \mathbf{B})}{\sqrt{2}}[\mathbf{B}^2 - \mathbf{E}^2 + I^2]^{1/2}, \quad (14)$$

$$\gamma = \frac{1}{\sqrt{2}} \left[1 + \frac{\mathbf{E}^2 + \mathbf{B}^2}{I^2} \right]^{1/2}. \quad (15)$$

To obtain a closed system of equations, one needs to account for the evolution of the physical fermion energy density ρ_χ . Incorporating energy conservation in an expanding universe, the equation for ρ_χ can be written phenomenologically as [44,49]

$$\partial_t \rho_\chi + 4\mathcal{H}\rho_\chi = \frac{1}{a^4}(\langle\sigma_E\rangle\langle E^2\rangle + \langle\sigma_B\rangle\langle \mathbf{E} \cdot \mathbf{B}\rangle), \quad (16)$$

where it is assumed that the plasma comprises relativistic particles possessing a statistically isotropic momentum distribution, $p_\chi = \rho_\chi/3$. It is worth noting that in Eq. (16) one could use $\langle\sigma_E E^2\rangle + \langle\sigma_B \mathbf{E} \cdot \mathbf{B}\rangle$. We defer a detailed comparison of different prescriptions for a subsequent publication.

IV. NUMERICAL SIMULATIONS

To determine the evolution of the system, we solve Eqs. (2)–(6) together with Eq. (16). This is done numerically on a lattice using the `Pencil` code [52]. For our choice $am_{\text{Pl}}/f = 60$, a grid of 512^3 points is sufficient. We start around $3e$ -folds before the end of inflation and initialize fields with the Bunch-Davies initial conditions. For simplicity, we choose a quadratic potential for the axion $V(\phi) = \frac{1}{2}m^2\phi^2$ with $m = 1.04 \times 10^{-6}m_{\text{Pl}}$. Even though this is observationally ruled out during inflation, it is a valid approximation during preheating and as such has been widely used in the literature [35–38]. We do not expect qualitative differences for more complicated potentials, like axion monodromy [28].

As outlined above, there are several possible descriptions of the Schwinger current. This is due to the nonperturbative nature of the effect and the fact that the solution is only known for a constant electric field limit. Thus, extrapolating from this to more realistic scenarios leads to different

prescriptions. We begin our analysis with the simplest parametrization: the collinear current description in the electric picture, given by Eq. (8), and compare it with the noncollinear formulation of Eqs. (10) and (11). We follow the definitions of Ref. [49] for the charge and the gauge-coupling constant. In the expression for the conductivities, we set $\mathcal{Q}^3 = 41/12$, which equals half the sum of the cubes of the hypercharges of all Standard Model Weyl fermions [while Eq. (7) refers to a single Dirac fermion]. The gauge-coupling constant is $e = g' = \sqrt{4\pi/137} \simeq 0.303$, but a realistic description of the Schwinger effect requires taking into account its running. Hence, in our simulations we use the gauge-coupling constant $e = g'(\tilde{\mu})$ defined as

$$g'(\tilde{\mu}) = \left([g'(m_Z)]^{-2} + \frac{41}{48\pi^2} \ln \frac{m_Z}{\tilde{\mu}} \right)^{-1/2}, \quad (17)$$

where $g'(m_Z) \simeq 0.35$, and $m_Z \simeq 91.2 \text{ GeV}$, with the characteristic energy scale $\tilde{\mu} = (\rho_E + \rho_B)^{1/4}$. The conductivities in Eqs. (8) and (11) depend on the electric and magnetic fields. In our numerical simulations, the conductivity is computed locally from the fields at each grid point. Nevertheless, defining it in terms of spatially averaged fields yields nearly identical results (detailed comparison will be presented in a subsequent publication).

We perform fully nonlinear simulations for the collinear [Eq. (8)] and noncollinear [Eqs. (10) and (11)] description of conductivities, consistently applying the local formulation in the equations of motion. For comparison, we also consider the evolution in the strong backreaction regime without fermions (see, e.g., [26,37]). Furthermore, we provide a comparison to the linear approximation with homogeneous inflaton dynamics, where $\mathbf{E} \cdot \mathbf{B}$ and $\nabla\phi$ are omitted in Eq. (2). The Hubble scale in the last case is taken to depend only on the inflaton $\mathcal{H}^2 = (8\pi/3m_{\text{Pl}}^2)a^2\rho_\phi$. The result of our simulations is shown in Fig. 1. In the top panel we show the evolution of the root-mean-square (rms) magnetic field strength, defined as $B_{\text{rms}}^2 = \int 4\pi k dk |B(k)|^2$, and the rms electric field E_{rms} , defined analogously. We provide this comparison for collinear and noncollinear cases.

We see that all cases operating in the large-coupling regime exhibit very similar suppression, regardless of the Schwinger current formulation and the inclusion of nonlinearities. This indicates that the presence of the Schwinger current prevents the gauge field from entering the strong backreaction regime, leading to a universal suppression. As a result, inhomogeneous inflaton gradient contributions remain suppressed, in contrast to studies neglecting the Schwinger effect, where such contributions become particularly important at the end of inflation [35,37,38]. Consequently, Schwinger-induced damping can be estimated within the linear approximation, allowing for the use of semianalytical methods.

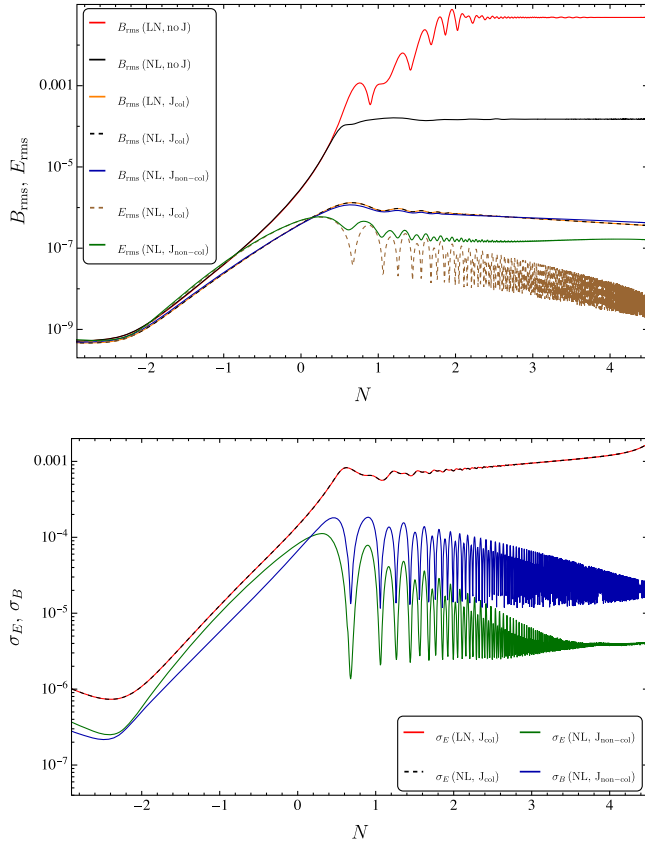


FIG. 1. Upper: the rms values of the comoving electric and magnetic fields under different assumptions. Magnetic fields are shown for the following cases: linear approximation with a homogeneous inflaton and no Schwinger current (red solid), nonlinear backreaction with a nonhomogeneous inflaton and no Schwinger current (black solid), collinear conductivity description in the linear (orange) and nonlinear (black dashed) regimes, and noncollinear conductivity (blue solid). Finally, the brown dashed and green solid curves are the rms electric fields that correspond to the black dashed and blue solid B_{rms} field results (collinear and noncollinear prescriptions, respectively). Lower: electric conductivity in the collinear description is shown for the linear case (red solid) and the nonlinear backreaction case (black dashed). The blue and green solid lines correspond to the noncollinear conductivity prescription. Color coding matches the upper panel.

On the lower panel of Fig. 1, we show the evolution of conductivities. We see that taking the collinear prescription, the electric conductivity is the same for the linear and nonlinear cases, whereas in the noncollinear prescription the electric and magnetic conductivities are oscillating in time.

V. UNIVERSALITY OF THE SCHWINGER BACKREACTION

It has been shown that the largest amplification of gauge fields during axion inflation occurs close to the end of inflation [28], since the tachyonic amplification depends on the axion velocity, which is maximal close to the end of

inflation. The growth of E and B can be described by a simple exponential growth rate (see [28] for a WKB analysis). Furthermore, both fields are almost equal during this growth. By examining the equation of motion for the gauge field [Eq. (3)], we see two competing terms: $(\alpha/f)(\partial_\tau\phi)B$ supports the tachyonic amplification, whereas the current $J = \sigma_E E$ opposes it. Initially, the tachyonic amplification term dominates and thus one of the two polarizations undergoes exponential enhancement. Since $E \approx B$, we can compare the two competing terms by comparing $(\alpha/f)\partial_\tau\phi$ to the conductivity σ_E . Using $\partial_\tau\phi = m_{\text{Pl}}\mathcal{H}\sqrt{\epsilon/4\pi}$ we get $(\alpha/f)\partial_\tau\phi \sim \mathcal{O}(100)\mathcal{H}$, where we took $\epsilon \sim 1$ close to the end of inflation and $\alpha m_{\text{Pl}}/f \sim 60\text{--}100$. For $\mathcal{H} \sim 10^{-5}m_{\text{Pl}}$ we see that the backreaction from Schwinger pair production occurs at $\sigma_E \sim 10^{-3}m_{\text{Pl}}$. We can also estimate the typical value of the electric and magnetic fields. First, we observe that $\coth(\pi E/B) \simeq 1$ for $E/B = \mathcal{O}(1)$. Furthermore, $eQ \simeq 1$, leading to $B \sim 6\pi^2(\alpha m_{\text{Pl}}/f)\mathcal{H}^2 \sim 10^{-6}m_{\text{Pl}}^2$.

Intriguingly, the above estimates for the conductivity and the value of the EM fields at the onset of Schwinger backreaction are consistently supported by our simulations. We thus uncover the existence of a universal behavior for axion inflation magnetogenesis, where the Schwinger effect is significant when gauge fields reach a value of $E, B \gtrsim \mathcal{O}(10^{-6})m_{\text{Pl}}$ close to the end of inflation. The Schwinger suppression will be less pronounced (largely irrelevant) for couplings that lead to weaker fields.

VI. CONSEQUENCES FOR MAGNETOGENESIS

The nondetection of secondary GeV photons from blazars provides indirect evidence for the presence of extragalactic magnetic fields in the intergalactic medium (possibly helical [53]). This observation motivates investigations into their origin in the early Universe. The prospect of magnetogenesis in axion inflation has been explored for couplings up to $\alpha m_{\text{Pl}}/f \leq 60$ [26] and, more recently, for $\alpha m_{\text{Pl}}/f = 75, 90$ [37]. These studies conclude that, for $\alpha m_{\text{Pl}}/f \geq 60$, the axion-U(1) inflation model is already marginally compatible with generating magnetic fields strong enough to account for the nonobservation of GeV photons in blazar spectra. Moreover, as we have seen, the Schwinger effect significantly reduces the final amplitude. To quantify the suppression, let us consider the present-day magnetic field strength and its coherence length L . Assuming that the inverse cascade process occurs almost immediately after inflation due to the existence of the Schwinger plasma (see the Appendix for details), the comoving magnetic field strength and correlation length follow $B^2 L = \text{const}$. Figure 2 shows that the inclusion of the Schwinger effect makes the resulting magnetic field largely independent of the coupling $\alpha/f \geq 60m_{\text{Pl}}$ and the prescription for the current (dynamical versus nondynamical; see Appendix) and well below the lower bound on

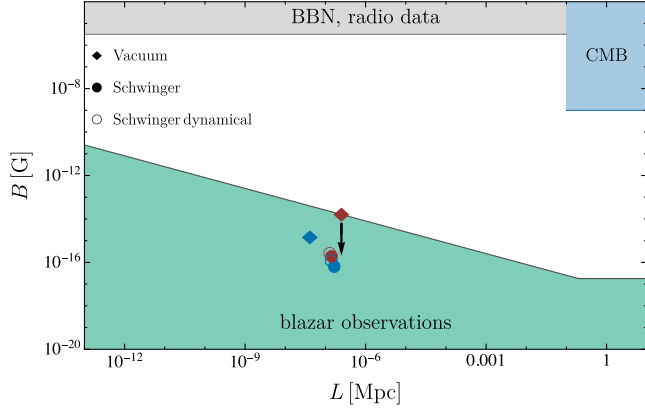


FIG. 2. Magnetic field strength as a function of coherence length for $am_{\text{Pl}}/f = 60, 90$ (red, blue, respectively) without Schwinger current (diamonds) and with a presence of a non-dynamical (filled circles) and dynamical (hollow circles) Schwinger current. The blue- and gray-shaded areas correspond to excluded upper bounds from observations of the cosmic microwave background (CMB) by Planck mission [54], big bang nucleosynthesis (BBN) [55], and radio data [56], and the green-shaded area indicates the lower bound on intergalactic magnetic field strength from blazar observations [57]. With the Schwinger current, the values lie below the blazar lower bound.

intergalactic magnetic field strength from blazar observations.

VII. HEAVY FERMION EFFECTS

So far, we have neglected the effects arising from finite fermion masses. From (7) it follows that a significant suppression of the Schwinger effect requires $\pi m^2 a^2 > e|Q|E$ or $m^2/E_{\text{ph}} \gtrsim \mathcal{O}(1)$, where E_{ph} is the amplitude of the physical electric field. We are focusing on the era close to the end of inflation, where $B_{\text{ph}}^2 \simeq E_{\text{ph}}^2 \sim 3H^2 m_{\text{Pl}}^2/8\pi$, or $E_{\text{ph}} \sim 0.1Hm_{\text{Pl}}$. The masses of Standard Model (SM) fermions are given by $m = yh$, where y is the Yukawa coupling and h is the Higgs VEV, which is expected to be nonzero during inflation, if the Higgs boson is a light field subject to de Sitter fluctuations. Among the electrically charged fermions, the electron has the smallest Yukawa coupling, $y_e \simeq 3 \times 10^{-6}$, making it the lightest. Thus, if electrons are too heavy to be efficiently produced via the Schwinger effect, all other charged fermions will be even more suppressed. To suppress the Schwinger effect, the fermion mass must satisfy $m_e^2 \gtrsim 0.1Hm_{\text{Pl}}$. Figure 3 shows how the presence of a large electron mass suppresses the Schwinger current and allows for the generation of a larger magnetic field. However, the constraint on m_s implies a lower bound on the Higgs VEV $h \gtrsim (0.3/y_e)\sqrt{Hm_{\text{Pl}}} \simeq 10^5 m_{\text{Pl}} \sqrt{H/m_{\text{Pl}}}$. To avoid super-Planckian values for the Higgs field, one requires a low inflationary scale: $H \lesssim 10^{-10} m_{\text{Pl}} \sim 10^9$ GeV. This provides a key result: a suppression of the Schwinger effect through

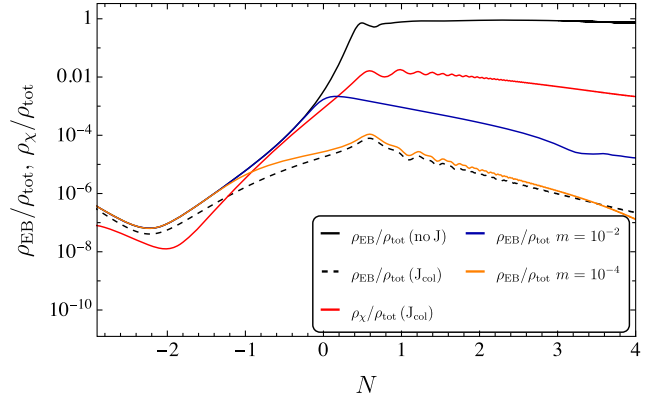


FIG. 3. The ratio of the EM energy density $\rho_{EB} \equiv \rho_E + \rho_B$ and the plasma energy density ρ_χ to the total energy density ρ_{tot} in the simulation box for $am_{\text{Pl}}/f = 60$. The black solid curve corresponds to a case without the Schwinger effect and the black dashed curve to a case with the Schwinger effect and massless fermions. The corresponding plasma energy density ratio is shown with red solid line. The EM energy density ratio for fermion masses $m = 10^{-4}, 10^{-2} m_{\text{Pl}}$ are shown in solid orange and blue, respectively.

fermion masses requires both low-scale inflation and large (possibly Planckian) field excursions for the Higgs boson. If one relies solely on de Sitter fluctuations to generate a Higgs field VEV, one expects $h \sim H\lambda^{-1/4}$, which leads to $H/m_{\text{Pl}} \gtrsim 10^{12}\sqrt{\lambda}$, where λ is the Higgs boson self-coupling constant. This requirement is difficult to satisfy given observational upper bounds on H , unless we consider an almost vanishing Higgs self-coupling. However, alternative mechanisms such as a direct coupling between the Higgs boson and the inflaton, or a nonminimal coupling to gravity can dynamically induce a large Higgs VEV during inflation.

These considerations point to an intriguing model-building challenge: any realistic suppression of the Schwinger effect involving SM fermions may lead to observable consequences at collider experiments through a modification of the Higgs boson sector. Before concluding, we must note that all simulations presented here refer to high-scale inflation, $H \sim 10^{-6} m_{\text{Pl}}$, and thus need to be redone for different Hubble scales. In particular, Fig. 3 shows that the Schwinger effect is still active if we consider large fermion masses, albeit weaker. That being said, reducing the scale of inflation allows for a large hierarchy between the Hubble scale and thus the possible electric field values are reduced, whereas the fermions can be equally heavy, if we take the Higgs VEV close to the Planck mass during inflation. Therefore, a definitive conclusion on the viability of axion inflation magnetogenesis requires a dedicated parameter scan, which we defer for future work.

VIII. SUMMARY AND OUTLOOK

We presented the first lattice simulations of the nonlinear evolution after axion inflation that self-consistently

incorporate Schwinger pair production. Our results demonstrate that the induced Schwinger current provides a robust backreaction that quenches gauge-field amplification once a universal critical conductivity $\sigma_E \sim 10^{-3} m_{\text{Pl}}$ and magnetic field strength $B_{\text{rms}} \sim 10^{-6} m_{\text{Pl}}^2$ are reached. These critical values are independent of the specific formulation of the current and mark the onset of plasma domination. We found that the Schwinger effect prevents the gauge field from entering the strong backreaction regime, keeping inflaton gradient contributions subdominant and enabling the use of semianalytical methods.

The resulting suppression of EM field production significantly reduces the amplitude of primordial magnetic fields, effectively ruling out high-scale axion inflation as a source of intergalactic magnetogenesis. Avoiding strong suppression requires fermion masses large enough to inhibit Schwinger production—necessitating a large Higgs VEV during inflation and favoring low-scale inflation scenarios. These findings motivate future studies on low-scale inflationary models and on connections between the axion and Higgs boson sectors, with potential implications for collider signatures. A different formulation of the Schwinger current, based on a numerical integration of $\partial_\tau J$ provides a very similar result for the present-day B field strength (see Fig. 2), which remains well below the level required to account for the blazar observations. The generation of inhomogeneities in fermion energy density and the resulting inhomogeneous bulk motions of the plasma remain suppressed during our simulations, as we show in the Appendix. However, addressing the full magnetohydrodynamic (MHD) evolution is an important task for future studies. Finally, the prevalence of the Schwinger effect in other areas of physics (e.g., [58,59]) provides an exciting avenue for applying the physical intuition gained and the code developed here to study these effects.

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DATA AVAILABILITY

The source code used for the simulations of this study, the *Pencil* code, is freely available from Refs. [52,60]. The simulation setups and the corresponding data are freely available from Ref. [61].

APPENDIX: CURRENT DYNAMICS, MAGNETOGENESIS, AND PLASMA MOTIONS

1. Evolution with different couplings

In this appendix, we demonstrate the evolution of the magnetic field and electric conductivities for coupling constants $am_{\text{Pl}}/f = 90$ (solid red), 75 (dashed black), and 60 (solid blue). For simplicity, we chose the collinear case for conductivities given by Eq. (8). As illustrated in Fig. 4, the impact of Schwinger suppression remains within an order-of-magnitude variation at strong coupling, highlighting the universality of the Schwinger backreaction.

2. Full integration of the current

Let us start by providing a heuristic derivation of the Schwinger current [41,62] in Minkowski space. We first compute the pair production rate [63] (for weak fields) $\Gamma \propto (eE)^2 e^{-\frac{\pi m^2}{eE}}$. These particles are accelerated by the electric field through $v = \frac{e}{m} \int E dt$. Combining these leads to an equation for the current as an integral over time or, equivalently, a first-order differential equation where dJ/dt is a function of E . Performing a more careful calculation in an expanding space-time similarly leads to the first-order differential equation for the dynamical current [43]

$$\partial_\tau J = \frac{(e|Q|)^3}{2\pi^2} E|B| \coth\left(\frac{\pi|B|}{E}\right), \quad (\text{A1})$$

where the fermion mass is dropped for simplicity and the fields are comoving as in the main text. One can integrate the above, by assuming constant (physical) EM fields and constant Hubble scale, and arrive at

$$J = \frac{(e|Q|)^3}{6\pi^2 \mathcal{H}} E|B| \coth\left(\frac{\pi|B|}{E}\right), \quad (\text{A2})$$

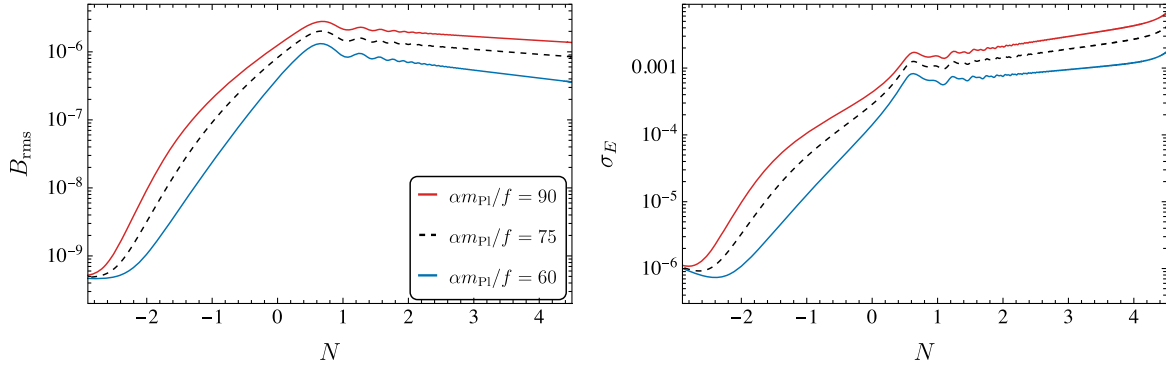


FIG. 4. Comparison of comoving B_{rms} and σ_E for cases with $am_{\text{Pl}}/f = 90, 75, 60$ (solid red, dashed black, solid blue curves) for the collinear case and full nonlinear evolution. The figure illustrates that Schwinger backreaction universally suppresses the magnetic field, with variations remaining within about an order of magnitude at strong coupling.

which is the form of the current that we used in our simulations and has been widely used in the literature [43–45,47,49].

Since we are evolving the full system on the lattice, there is no added complication in numerically integrating Eq. (A1) instead of using Eq. (A2). Figure 5 shows the results of a simulation run, where the axion and EM fields are computed at each point on the lattice, while the current is computed by integrating Eq. (A2), similarly at each point on the lattice. From the top panel of Fig. 5, we observe that the evolution of the physical electric and magnetic fields, as well as the current, is similar for both dynamical and nondynamical current prescriptions. At the end of inflation, however, the dynamical current has an amplitude roughly twice that of the nondynamical one. Although the dynamical field is somewhat larger, its amplitude is still too small to produce a significant effect. In the next section (Magnetogenesis), we show that this difference leads to almost indistinguishable present-day magnetic fields.

The middle panel of Fig. 5 displays the evolution of the comoving electric and magnetic fields and the comoving current. It is worth noting the difference in the late-time evolution of comoving electric and magnetic fields for the dynamical and nondynamical prescriptions. In Magnetogenesis, we discuss in detail that, for the late-time evolution, the crucial aspect is conservation of magnetic helicity, where the evolution of the magnetic field is tied to the coherence length (such that the evolution of the magnetic field should always be considered together with that of the coherence length). We show that the mean magnetic helicity density is approximately conserved for both treatments of the Schwinger current, dynamical and nondynamical (see Fig. 9). In particular, for the dynamical current, after the end of inflation, the magnetic field increases while the coherence length decreases, with the two effects compensating each other so that the mean magnetic helicity density remains approximately constant. This allows us to assume that the inverse cascade operates from around the end of inflation for both cases. We

emphasize that, within this treatment, the dominant contribution for the present-day values comes from around the peak of both the magnetic field and the coherence length during their generation (see Fig. 6 and the discussion in the Magnetogenesis section).

The bottom-left panel of Fig. 5 compares the total energy density in the absence of the Schwinger effect (vacuum case, solid black) with the cases including Schwinger current, for both dynamical (blue) and nondynamical (dashed black) prescriptions. The energy density evolves similarly in both cases, with slightly weaker suppression from Schwinger backreaction in the dynamical case. The bottom-right panel shows the evolution of the conductivities. We find that the electric conductivity σ_E evolves very similarly in both cases, which explains the comparable suppression of the magnetic fields.

3. Magnetogenesis

This section presents the computation details of the present-day magnetic field. The root-mean-square value of the physical magnetic field and its comoving coherence length are given by

$$B_{\text{rms}}^{\text{ph}} = \frac{1}{a^2} \sqrt{\int d\log k \cdot P_B(k)}, \quad L_c = \frac{\int d\log k \cdot k^{-1} \cdot P_B}{\int d\log k \cdot P_B}. \quad (\text{A3})$$

After accounting for the nonlinear evolution of the fields, the magnetic field strength and its coherence length at the present epoch are given by (see [37] for details)

$$B_{\text{rms}}^{\text{ph}}|_0 = 9.2 \times 10^{-15} \text{ G} \sqrt{\frac{\int d\log k \cdot P_B}{\rho_{\text{tot}}}} \left(\frac{10^{-6} m_{\text{Pl}}}{H} \right) r_A^{1/3},$$

$$L_c|_0 = 0.8 \text{ pc} (\mathcal{H} L_c) \left(\frac{10^{-6} m_{\text{Pl}}}{H} \right) r_A^{-2/3}, \quad (\text{A4})$$

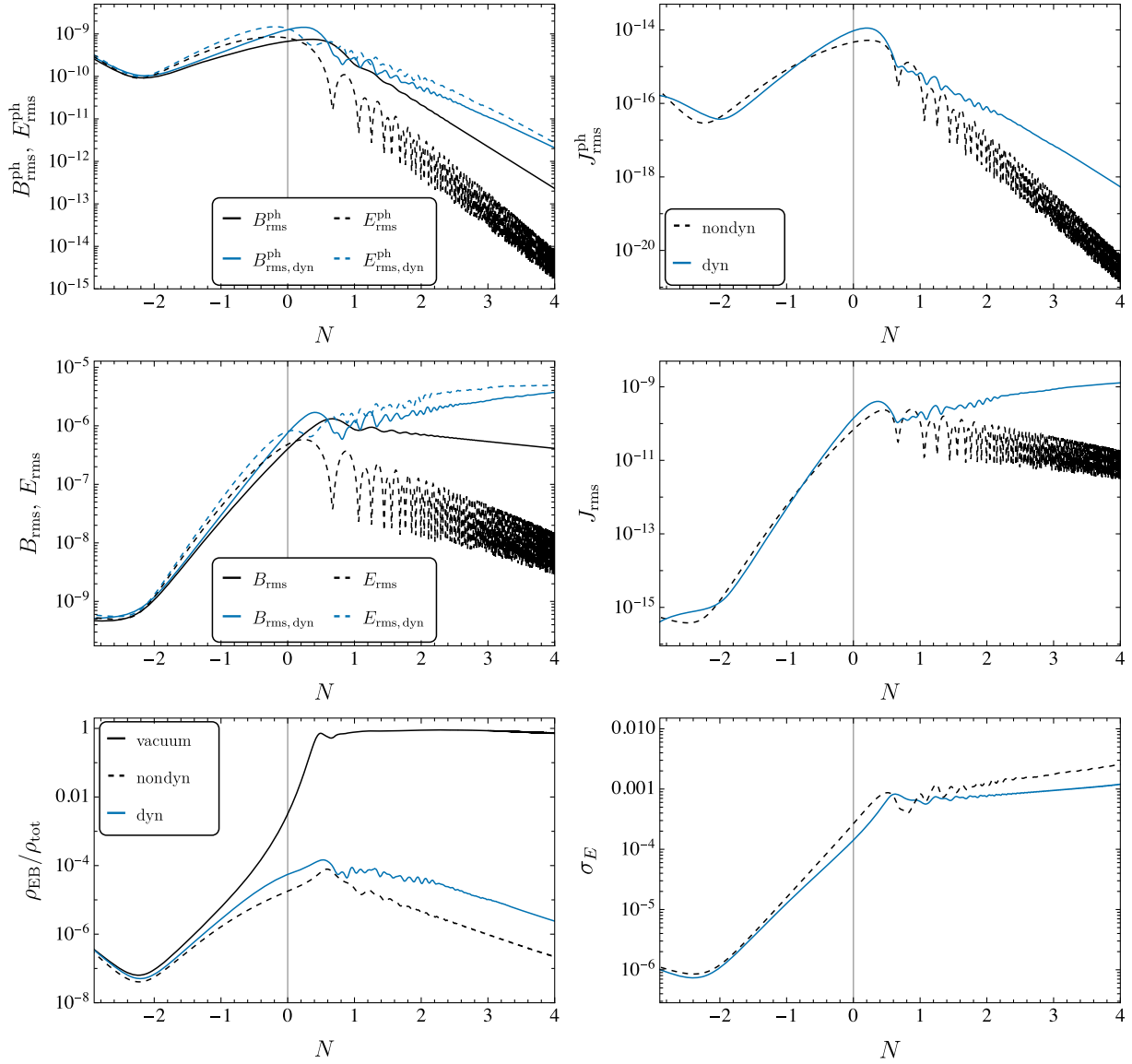


FIG. 5. Comparison of physical $B_{\text{rms}}^{\text{ph}}$, $E_{\text{rms}}^{\text{ph}}$, and $J_{\text{rms}}^{\text{ph}}$ and comoving E_{rms} , B_{rms} , J_{rms} , $\rho_{\text{EB}}/\rho_{\text{tot}}$, σ_E for cases with dynamical and nondynamical current. The axion-gauge coupling is chosen as $\alpha m_{\text{Pl}}/f = 60$. In the upper-left panel, black curves (solid for $B_{\text{rms}}^{\text{ph}}$, dashed for $E_{\text{rms}}^{\text{ph}}$) show the fields with a nondynamical current, while blue curves (solid for $B_{\text{rms}}^{\text{ph}}$, dashed for $E_{\text{rms}}^{\text{ph}}$) show the fields with a dynamical current. The middle panel is similar to the top panel, but it displays comoving fields $B_{\text{rms}} = a^2 B_{\text{rms}}^{\text{ph}}$, $E_{\text{rms}} = a^2 E_{\text{rms}}^{\text{ph}}$, and $J_{\text{rms}} = a^3 J_{\text{rms}}^{\text{ph}}$. In the lower-left panel, the solid black curve shows the EM energy density for the vacuum case (without Schwinger suppression), the dashed blue curve is for the Schwinger case with a nondynamical current, and the solid blue curve is for the case with a dynamical current. The same color coding applies to the upper-right, middle-right, and lower-right panels showing comoving J_{rms} and σ_E . The vertical gray line indicates the end of inflation.

where $r_A = \max(1, \mathcal{H}L_c/v_A)$ and $v_A = \sqrt{B^2/(\rho_\chi + p_\chi)}$ is the Alfvén velocity that characterizes the propagation speed of MHD disturbances in a plasma, where we assume $p_\chi = \rho_\chi/3$. The definition of the Alfvén velocity depends critically on the presence of a conducting medium. Without the Schwinger effect, the Universe close to the end of inflation is essentially vacuumlike, and the Standard Model plasma only forms after the end of inflation through

reheating via the decay of the inflaton field into relativistic particles. In this case, since there is no significant plasma yet, the Alfvén velocity is ill defined. For axion inflation, however, reheating can be effectively instantaneous, which allows for a conducting plasma to form quickly and for the Alfvén velocity to become meaningful.

Including the Schwinger effect further changes this picture: the strong gauge fields generated during axion inflation can produce electron-positron pairs via

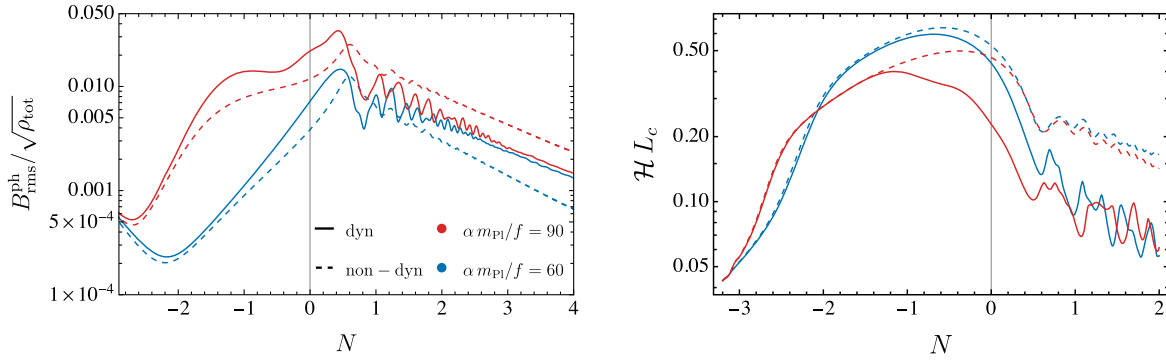


FIG. 6. The magnetic field strength normalized by the total energy density (left) and the coherence length normalized to the comoving Hubble scale (right) for dynamical (solid) and nondynamical (dashed) currents. Red curves correspond to $\alpha m_{\text{Pl}}/f = 90$ and blue curves to $\alpha m_{\text{Pl}}/f = 60$. The vertical gray line indicates the end of inflation. The color coding is the same for both panels. The dynamical current prescription yields a dimensionless magnetic field strength that is approximately twice as large at the end of inflation, but with a shorter coherence length. Taking into account the nonlinear processing via Eq. (A4), this leads to nearly indistinguishable present-day magnetic fields, which are about 2 orders of magnitude lower than the lower bound inferred from blazar observations. Figure 2 illustrates how these results compare with current observational constraints.

nonperturbative particle creation. This process generates a conducting plasma well before the conventional reheating phase, providing a medium in which MHD waves can propagate. Consequently, the Alfvén velocity becomes well defined earlier, and its value reflects the energy density of the newly created plasma. This distinction is crucial for accurately modeling magnetic field evolution: neglecting the Schwinger effect leads to an incorrect estimate of the plasma contribution, which, in turn, miscalculates the Alfvén velocity and the resulting magnetogenesis dynamics in the early Universe.

Figure 6 shows the evolution of the dimensionless magnetic field strength relative to the total energy density and the coherence length normalized to the comoving Hubble scale. The dynamical current produces a dimensionless magnetic field strength approximately twice as large at the end of inflation, but with a smaller coherence length. After accounting for the nonlinear evolution of the fields using Eq. (A4), we compute the resulting present-day magnetic field strength and coherence length. It is clear from Fig. 6 that the dominant contribution comes from around the end of inflation. This is because both the amplitude and the coherence length entering Eq. (A4) are dominated by their peak values, which occur around the end of inflation. Figure 2 compares these results with observational constraints. Because of the universal suppression from the Schwinger backreaction, the results for both coupling constants $\alpha m_{\text{Pl}}/f = 90$ and $\alpha m_{\text{Pl}}/f = 60$ are very close to each other, see Table I. Regardless of the Schwinger current description, the resulting magnetic field strength is suppressed by roughly 2 orders of magnitude, effectively ruling out magnetogenesis in axion inflation. We also compute the magnetic field strength one, two, and three e -folds after the end of inflation for both the dynamical and nondynamical current descriptions. The

results are shown in Fig. 7, with the corresponding numerical values listed in Table II. We find that the dynamical current produces magnetic fields with slightly shorter coherence lengths, consistent with the coherence length evolution shown in Fig. 6. However, the final field amplitudes are comparable in the two cases, while still lying far below the values indicated by blazar observations.

It is interesting to consider the effect of the Schwinger backreaction on the helicity of the produced magnetic fields. The magnetic energy and magnetic helicity can be expressed as integrals of the corresponding energy and

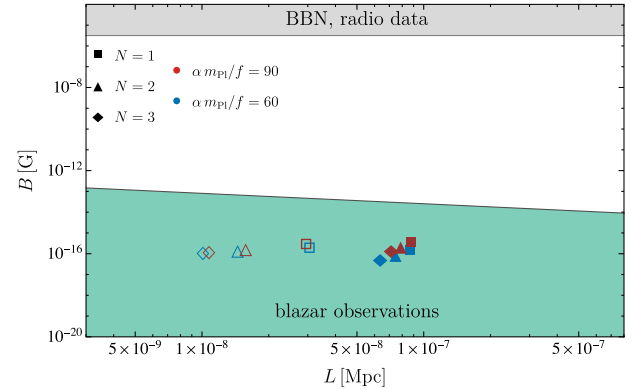


FIG. 7. Present-day magnetic field strengths and corresponding coherence lengths evaluated one (squares), two (triangles), and three (diamonds) e -folds after the end of inflation. Filled symbols correspond to the nondynamical Schwinger current prescription, while open symbols correspond to the dynamical Schwinger current prescription. Red symbols denote $\alpha m_{\text{Pl}}/f = 90$, and blue symbols denote $\alpha m_{\text{Pl}}/f = 60$. We find that the dynamical current generates magnetic fields with smaller coherence lengths. For both current prescriptions, the resulting present-day magnetic field strengths are comparable, but remain well below the lower limits inferred from blazar observations.

TABLE I. Comparison of the present-day magnetic field strength $B_{\text{rms}}^{\text{ph}}|_0$ and coherence length $L_c|_0$ for coupling constants $\alpha m_{\text{Pl}}/f = 90, 60$, considering the case without the Schwinger effect (vacuum case) and with the Schwinger effect using both nondynamical and dynamical current prescriptions. The values are computed from the power spectrum at the end of inflation. Reference values for the vacuum case are taken from [37]. The observational bound is given by $B_{\text{bound}} = 1.8 \times 10^{-17} \text{ G} (L_c/0.2 \text{ Mpc})^{-1/2}$ [57]. For a coherence length $L_c = 10^{-7} \text{ Mpc}$, this corresponds to $B_{\text{bound}} = 2.55 \times 10^{-14} \text{ G}$. The results including the Schwinger effect are at least 2 orders of magnitude below this bound, even at strong coupling and regardless of the current description.

$\alpha m_{\text{Pl}}/f$	$B_{\text{rms}}^{\text{ph}} _0$ (vac) (G)	$B_{\text{rms}}^{\text{ph}} _0$ (nondyn) (G)	$B_{\text{rms}}^{\text{ph}} _0$ (dyn) (G)	$L_c _0$ (vac) (Mpc)	$L_c _0$ (nondyn) (Mpc)	$L_c _0$ (dyn) (Mpc)
60	1.7×10^{-15}	7.25×10^{-17}	1.35×10^{-16}	4.1×10^{-8}	1.65×10^{-7}	1.38×10^{-7}
90	1.9×10^{-14}	2.15×10^{-16}	2.92×10^{-16}	2.5×10^{-7}	1.43×10^{-7}	1.28×10^{-7}

TABLE II. Values for the present-day magnetic field strength and coherence length evaluated at one, two, and three e -folds after the end of inflation for the nondynamical and dynamical current prescriptions and coupling constants $\alpha m_{\text{Pl}}/f = 60$ and $\alpha m_{\text{Pl}}/f = 90$.

N	$\alpha m_{\text{Pl}}/f$	$B_{\text{rms}}^{\text{ph}} _0$ (nondyn) (G)	$B_{\text{rms}}^{\text{ph}} _0$ (dyn) (G)	$L_c _0$ (nondyn) (Mpc)	$L_c _0$ (dyn) (Mpc)
1	60	1.39×10^{-16}	1.8×10^{-16}	8.68×10^{-8}	3.05×10^{-8}
	90	3.3×10^{-16}	2.73×10^{-16}	8.8×10^{-8}	2.94×10^{-8}
2	60	9.4×10^{-17}	1.56×10^{-16}	7.46×10^{-8}	1.45×10^{-8}
	90	2.34×10^{-16}	1.85×10^{-16}	7.86×10^{-8}	1.57×10^{-8}
3	60	5.65×10^{-17}	1.26×10^{-16}	6.36×10^{-8}	1.01×10^{-8}
	90	1.51×10^{-16}	1.38×10^{-16}	7.12×10^{-8}	1.08×10^{-8}

helicity densities in both position and momentum space [26]

$$E_B(t) = \frac{1}{V} \int_V d^3x \frac{\langle B^2 \rangle}{2} = \int_0^\infty dk \mathcal{E}_B(k, t), \quad (\text{A5})$$

$$H_B(t) = \frac{1}{V} \int_V d^3x \langle \mathbf{A} \cdot \mathbf{B} \rangle = \int_0^\infty dk \mathcal{H}_B(k, t). \quad (\text{A6})$$

It is useful to define the integrated helicity fraction as

$$h_B = \frac{H_B}{2L_c E_B}, \quad (\text{A7})$$

which is invariant under the transformation between physical and comoving fields. By construction, the value of the integrated helicity fraction is between 0 and 1 in the absolute values, with the equality holding for maximally helical fields. In the absence of the Schwinger effect and for small axion-gauge couplings, the resulting magnetic power spectrum is effectively maximally helical. This occurs because the gauge-field mode amplified during inflation grows by several orders of magnitude more than the mode that is amplified only during preheating. In the intermediate-to-large-coupling regime, however, rescattering effects between the gauge and axion modes become important. In this case, the subdominant helicity is generated through the scattering of the dominant mode off the axion field. This process mainly affects shorter wavelength modes (large wave numbers), leading to a nearly nonhelical field on small scales, while the large-scale modes produced during inflation remain significantly helical [26,28].

The situation changes in the presence of the Schwinger effect. As discussed in the main part of the paper, the Schwinger effect suppresses nonlinear backreaction, preventing rescattering between the axion and gauge fields. As a result, the magnetic field remains maximally helical throughout reheating, as illustrated in Fig. 8. We show

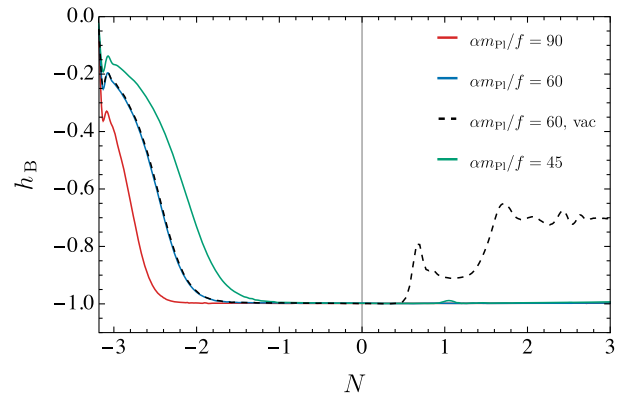


FIG. 8. The integrated helicity fraction as a function of the number of e -folds for $\alpha m_{\text{Pl}}/f = 90$ (red), $\alpha m_{\text{Pl}}/f = 60$ (blue), and $\alpha m_{\text{Pl}}/f = 45$ (green), including the Schwinger effect. The black dashed curve shows $\alpha m_{\text{Pl}}/f = 60$ for the vacuum case (without the Schwinger effect). The vertical gray line indicates the end of inflation. With the Schwinger effect, nonlinear backreaction and rescattering are suppressed, so the subdominant helicity mode is not enhanced, and the magnetic field remains maximally helical throughout reheating. By contrast, in the vacuum case, rescattering generates the opposite helicity, reducing the integrated helicity fraction to about 0.7 in the absolute value.

the integrated helicity fraction as a function of time for evolution with the Schwinger effect at couplings $\alpha m_{\text{Pl}}/f = 90$ (red), $\alpha m_{\text{Pl}}/f = 60$ (blue), and $\alpha m_{\text{Pl}}/f = 45$ (green). In all cases, the magnetic field becomes fully helical shortly after the start of the simulation and retains its maximal helicity until the end of the simulation. For comparison, we also show the evolution with $\alpha m_{\text{Pl}}/f = 60$ (black dashed) in the vacuum case (without the Schwinger effect). Here, the magnetic field initially becomes helical, but strong nonlinear backreaction and rescattering subsequently generate the growth of opposite polarization, reducing the integrated helicity fraction to an absolute value of approximately 0.7. This difference is crucial for the subsequent evolution of the magnetic fields [64].

Another crucial aspect is the conservation of magnetic helicity. The helicity density is approximately conserved [65,66]

$$H_B = h_B B_{\text{rms}}^2 L_c \approx \text{const.} \quad (\text{A8})$$

As a result, helicity conservation imposes a nontrivial constraint on the subsequent evolution of the magnetic field. In particular, as the system evolves and magnetic energy is dissipated, the coherence length must increase in order to maintain approximately constant helicity. This leads to an inverse transfer of power from small to large scales, with B_{rms}^2 decreasing while L_c grows such that their product remains nearly constant. Consequently, even if magnetic fields are initially generated on small scales, helicity conservation enables the efficient buildup of large-scale coherent fields at later times. We show the evolution of the helicity density in Fig. 9 for two coupling constants $\alpha m_{\text{Pl}}/f = 90$ (red), $\alpha m_{\text{Pl}}/f = 60$ (blue), and for both dynamical (solid) and nondynamical (dashed) treatments of the Schwinger current. We see that initially the helicity fraction grows as expected, but after the end of inflation it is

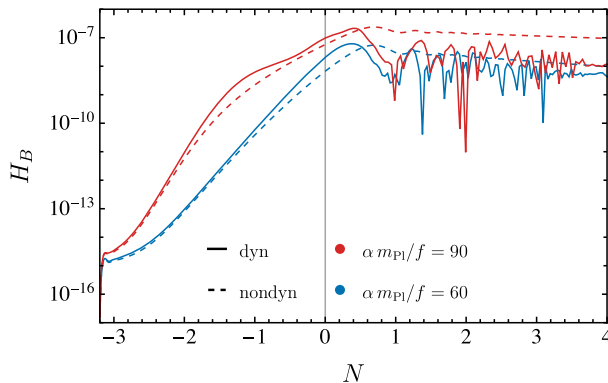


FIG. 9. The evolution of the magnetic helicity density (absolute value) expressed in comoving fields for $\alpha m_{\text{Pl}}/f = 90$ (red), $\alpha m_{\text{Pl}}/f = 60$ (blue), and for both dynamical (solid) and nondynamical (dashed) treatments of the Schwinger current. We see that the helicity density is approximately constant after the end of inflation for both dynamical and nondynamical cases.

approximately conserved for both treatments of the Schwinger current.

We do not expect exact conservation of helicity, since the system is not in the ideal MHD limit. In particular, finite conductivity, the presence of Schwinger pair production, and the inflaton introduce nonideal effects that can lead to helicity dissipation or increase. Nevertheless, the approximate conservation observed here indicates that these effects are not efficient enough to significantly alter the helicity budget at late times. This suggests that the subsequent evolution of the magnetic field may proceed via an inverse transfer, with a gradual increase of the coherence length while maintaining nearly constant $B_{\text{rms}}^2 L_c$, as assumed in equations for magnetic field evolution (A4).

We must stress here that, even though we are able to follow the simulation of the inflaton-photon-Schwinger plasma system for several e -folds after the end of inflation, simulation does not include the MHD effects. In the next section, we outline possible implications of them.

4. Inhomogeneous energy density and bulk motions

For completeness, we also note that the Schwinger pair plasma will not remain at rest, but it can be accelerated by the Lorentz force, $\mathbf{J} \times \mathbf{B}$. This leads to some of the EM energy being converted into kinetic energy at the rate $\langle \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) \rangle$, where $\mathbf{u}$ is the bulk velocity of the plasma [67]. A useful measure of the importance of nonlinear effects is the magnetic Reynolds number. However, in the absence of a well-defined bulk flow velocity, this quantity cannot be meaningfully constructed. Instead, the natural velocity scale in the system is the Alfvén speed v_A , and the relevant dimensionless parameter becomes the Lundquist number

$$\text{Lu} = v_A L_c \sigma_E, \quad (\text{A9})$$

which plays an analogous role in characterizing the relative importance of advection and diffusion of the magnetic field. When the Lundquist number becomes large, we can no longer ignore the production of inhomogeneities and the resulting inhomogeneous bulk motions of the plasma. The evolution of the Lu number is shown in Fig. 10 for three values of the coupling strength $\alpha m_{\text{Pl}}/f$. Evidently, for large axion-gauge couplings, nonlinearity and hydromagnetic effects can become important. However, we observe that the Lundquist number exceeds unity only near or after the end of inflation. This would mean that part of the generated EM energy is being converted into kinetic energy, which might diminish the magnetic field even further. Consequently, primordial magnetogenesis from axion inflation can become even more challenging, leaving the main conclusion of our paper intact.

The physical effect of bulk motions is that they affect the EM field and reorganize it to form larger-scale structures while preserving the magnetic helicity. The resulting

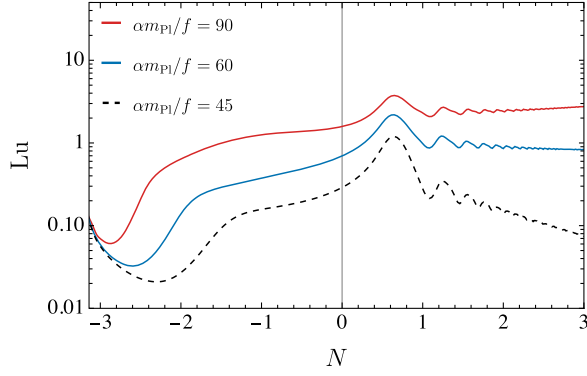


FIG. 10. The Lundquist number $\text{Lu} = v_A L_c \sigma_E$ versus the number of e -folds N for $\alpha m_{\text{Pl}}/f = 90$ (red), 60 (blue), and 45 (black dashed). The vertical gray line indicates the end of inflation. The Lundquist number exceeds unity only very close to or after the end of inflation, suggesting that any onset of plasma motion may become relevant at this late stage.

energy spectra then exhibit a phenomenon known as an inverse cascade [68]. To account for the resulting inhomogeneities and the bulk motions of plasma, Eq. (16) must be amended to include extra divergence terms that would vanish after volume averaging. Phenomenologically, the governing equations for the comoving fermion energy density, $\tilde{\rho}_\chi = a^4 \rho_{\text{ph},\chi}$, take the form

$$\partial_\tau \tilde{\rho}_\chi + \nabla \cdot \left(\frac{4}{3} \tilde{\rho}_\chi \mathbf{u} \right) = \mathbf{J} \cdot \mathbf{E} + 2\tilde{\rho}_\chi \nu \mathbf{S}^2 + \nabla \cdot (\kappa \nabla \tilde{\rho}_\chi), \quad (\text{A10})$$

where $\mathbf{u}$ is the velocity of the bulk motions of fermions, which obeys the usual momentum equation

$$\frac{4}{3} \tilde{\rho}_\chi (\partial_\tau \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) = -\frac{1}{3} (\nabla \tilde{\rho}_\chi + \mathbf{u} \partial_\tau \tilde{\rho}_\chi) + \mathbf{J} \times \mathbf{B} + \nabla \cdot (2\tilde{\rho}_\chi \nu \mathbf{S}), \quad (\text{A11})$$

where ν is the viscosity and $\mathbf{S}$ is the rate-of-strain tensor with the components $S_{ij} = (\partial_i u_j + \partial_j u_i)/2 - \delta_{ij} \partial_k u_k/3$. The term $2\tilde{\rho}_\chi \nu \mathbf{S}^2$ on the right-hand side of Eq. (A10) denotes viscous heating and κ is a radiative diffusion coefficient, which we expect to be comparable to the viscosity. Since we work here with comoving fermion energy density, the $4\mathcal{H}\rho_\chi$ term from Eq. (16) has disappeared.

The bulk motions also affect the current density, which then obeys the equation

$$\mathbf{J} = \sigma_E (\mathbf{E} + \mathbf{u} \times \mathbf{B}) + \sigma_B \mathbf{B}. \quad (\text{A12})$$

The $\mathbf{u} \times \mathbf{B}$ term is the electromotive force and leads to a feedback of the fermion velocity onto the EM field. The nonlinear interaction between $\mathbf{u}$ and $\mathbf{B}$ leads to a turbulent cascade of energy that is finally dissipated through the first two terms on the right-hand side of Eq. (A10); see Ref. [69] for earlier work on these aspects. The detailed investigation of the resulting turbulence is, however, beyond the scope of the present paper and presents a fruitful avenue for future work.

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