

Dynamo mechanisms

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Abstract. Dynamo theory is reviewed with particular emphasis on recent developments. There now seems to be a strong case for dynamo effects that are driven by the magnetic field itself. This is linked to recent interpretations of the observed stellar cycle periods which suggest that the ratio of cycle frequency to rotational frequency increases, up to some point, with stellar chromospheric activity. This ratio may be interpreted as a measure of the alpha-effect in dynamo theory, which would then increase with magnetic field strength. The important question of what happens as stars become fully convective is also addressed. It is argued that the dynamo does not work at the bottom of the convection zone, but in the entire convection zone proper. However, because of turbulent pumping effects the magnetic field is pushed to the bottom of the convection zone or, in the case of a fully convective star, to its center.

1. Introduction

Stellar chromospheric and coronal activity is usually explained by some kind of dynamo process, which converts kinetic energy into magnetic energy. For example, for turbulent convection at sufficiently high magnetic Reynolds numbers (small enough magnetic diffusivity) small scale magnetic fields are produced (Meneguzzi & Pouquet 1989, Nordlund et al. 1992, Brandenburg et al. 1996). However, many stars show cyclic behavior. Explaining such behavior requires some extra ingredients, such as rotation, shear, and vertical density stratification (e.g. Moffatt 1978). Those extra ingredients tend to give the flow some swirl and make it helical—just like cyclones. The shear from *differential* rotation tends to align the field with the toroidal direction, converting poloidal magnetic field into toroidal. To close the cycle, poloidal magnetic field is generated from toroidal via cyclonic convection.

Fig. 1 shows how, in the standard picture, rotation twists a rising flux loop such that an extra *turbulent* electric field is induced (the turbulent electromotive force \mathcal{E}), which has a component perpendicular to the mean magnetic field, i.e. $\mathcal{E} = \alpha \langle \mathbf{B} \rangle$. Note also that descending convective elements contract. They too produce a field-aligned current in the same direction; see Fig. 2.

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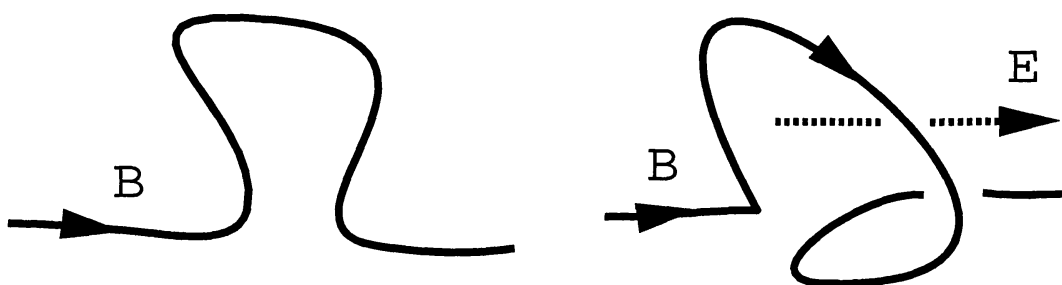


Figure 1. Convective element expands as it ascends. Rotation causes the flow to swirl creating a loop that induces a field-aligned current.

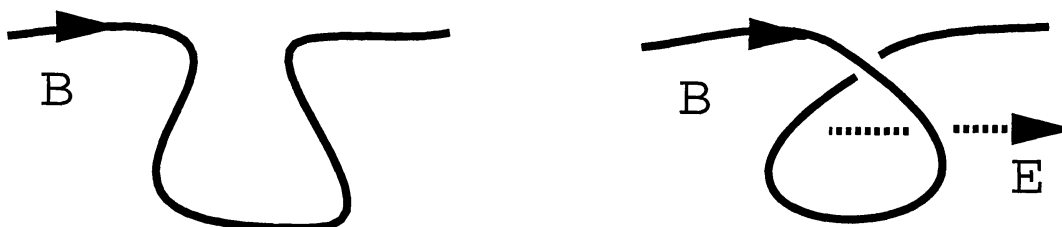


Figure 2. Convective element contracts as it descends. Again, rotation causes the flow to swirl creating a loop that induces a field-aligned current.

On the southern hemisphere α must however have the opposite sign, because there the rotation vector points in the opposite direction. This is also the reason why the swirl of cyclones and anticyclones is reversed on the southern globe.

The α parameter gives rise to exponentially growing solutions of the induction equation provided the magnitude of α is large enough (large enough dynamo number). If this α -effect is supplemented by differential rotation (the Ω -effect), one talks about an $\alpha - \Omega$ dynamo (e.g., Parker 1979, Krause & Rädler 1980). Figure 3 gives a qualitative explanation of why this α -effect can lead to dynamo waves propagating in the toroidal direction. Consider first some pre-existing poloidal magnetic field loop (Fig. 3a). In lower latitudes the deeper regions of the sun spin slower, giving rise to a toroidal field as shown in Fig. 3b. This toroidal field induces a current and the α -effect produces a new magnetic field parallel to it (Fig. 3c).

Comparing Figs. 3a and 3c we notice the emergence of a new loop near the equator with *opposite* orientation relative to the original loop in Fig. 3a. The larger loop in 3c, however, has the same orientation as the loop in Fig. 3a, which therefore appears to have migrated away from the equator. (Three further applications of shear and α -effect bring the situation full circle back to the configuration in Fig. 3a.) The field migration seen in this model is of course in the “wrong” direction, because in the sun the sunspot belts migrate equatorward. This is known as the dynamo dilemma (Parker 1987). In the early days of dynamo theory, before helioseismology told us otherwise, one used to believe that the sun spun faster in deeper layers than at the surface. In that case the dynamo waves would go in the right direction. There are some indications supporting this possibility in the case of accretion discs (Brandenburg & Donner 1997), but it is not clear that similar circumstances apply to the case of stars. An opposite

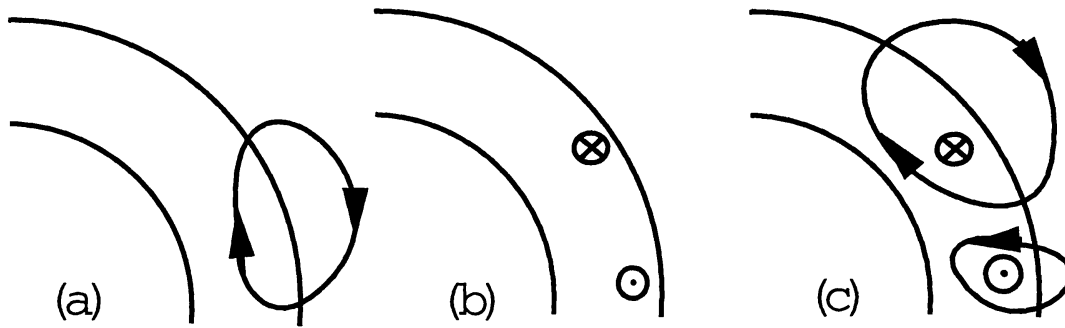


Figure 3. A poloidal magnetic field loop is being sheared out by differential rotation causing toroidal fields. The α -effect produces new poloidal field loops, but shifted polewards.

angular velocity gradient would indeed be consistent with the observation that very young sunspots have faster proper motion than older spots. (There is now no good explanation for this property of sunspot proper motions.)

If the sign of α was for some reason reversed (negative in the northern hemisphere) then this would turn the dynamo wave into the right direction. Another possibility that has been discussed already by Parker (1987), and more recently by Durney (1996) and Choudhuri, Schüssler, & Dikpati (1995) is to invoke meridional circulation to turn the dynamo wave around. The most recent model of that type is by Dikpati & Charbonneau (1999), where a more realistic profile of differential rotation has been adopted.

2. Stellar cycles

The induction equation is linear in the magnetic field. Thus, the field would continue to increase unless there is some nonlinear feedback mechanism. One thought therefore that α may be quenched as the magnetic field becomes dynamically important, i.e. when magnetic energy and kinetic energy of the turbulence become similar. In a recent paper, Brandenburg, Saar, & Turpin (1998) have suggested that α may actually *increase* with magnetic field strength—up to some limit anyway. This idea is based on a recent plot of observed stellar cycle frequencies, $\omega_{\text{cyc}} = 2\pi/P_{\text{cyc}}$ (normalized by the rotational frequency, $\Omega = 2\pi/P_{\text{rot}}$) versus the mean chromospheric activity parameter, $\langle R'_{\text{HK}} \rangle$; see Fig. 4.

In Fig. 4 there is a clear segregation into two branches with active and inactive stars (denoted by numbers and letters, respectively). On both branches $\omega_{\text{cyc}}/\Omega$ is an increasing function of $\langle R'_{\text{HK}} \rangle$. Using $\alpha - \Omega$ dynamo theory one finds that

$$\omega_{\text{cyc}} \approx \sqrt{\alpha \Omega'}, \quad (1)$$

where $\Omega' = \partial\Omega/\partial r$ is the radial Ω -gradient. The nondimensional cycle frequency can be written as

$$\frac{\omega_{\text{cyc}}}{\Omega} \approx \left(\frac{\alpha}{\Omega H} \right)^{1/2} \left(\frac{H \Omega'}{\Omega} \right)^{1/2}, \quad (2)$$

where H is the typical density scale height in the lower part of the convection zone. Now if α depends on field strength, i.e. $\alpha = \alpha(\langle \mathbf{B} \rangle)$, then we can use

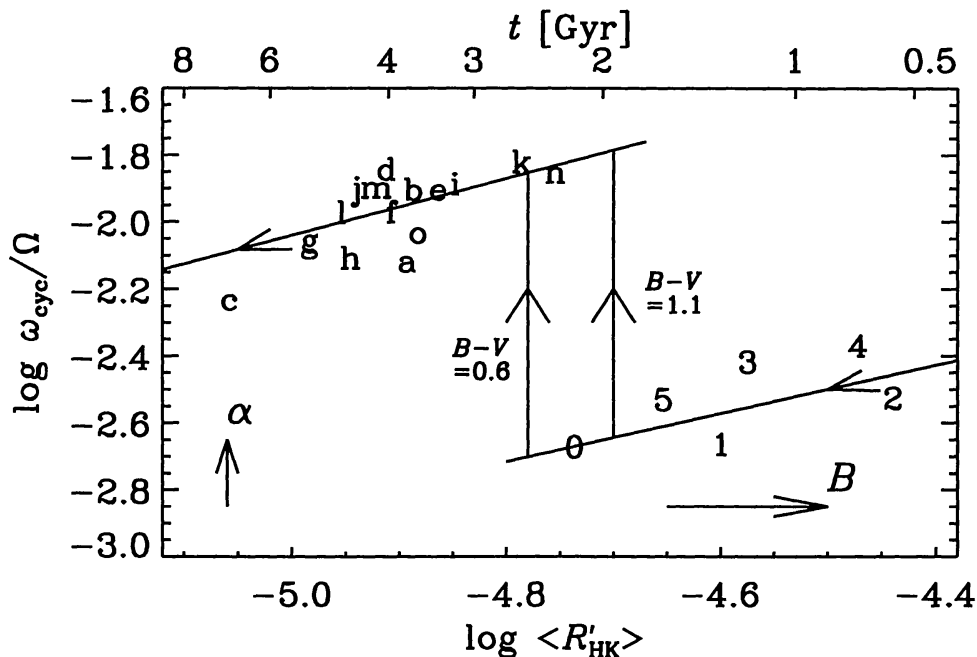


Figure 4. Ratio of cycle frequency to rotational frequency versus activity parameter $\langle R'_{HK} \rangle$, which in turn is related to the star's age. Note the increase in the frequency ratio as the activity goes up. [Adapted from Brandenburg, Saar, & Turpin (1998).]

$(\omega_{cyc}/\Omega)^2$ as a measure of $\alpha/\Omega H$. But this is of course quite unusual, because it would suggest that α is now an *increasing* function of field strength (Fig. 5a), not quenched (Fig. 5b), as is usually assumed. For details of this model see Brandenburg, Saar, & Turpin (1998) and Brandenburg (1998a).

It should however be pointed out that this result is model dependent. In fact, the conclusion to be drawn from this model applies only to the case of the one mode truncation. In one or two dimensional models with a continuum of wavenumbers the observed behavior can no longer be reproduced, even for strong anti-quenching of α . This may seem rather alarming. However, it should be remembered that while $\alpha - \Omega$ dynamo theory seems to reproduce many observed features at least qualitatively, some details of the theory may not be correct. For example, the product in the expression $\mathcal{E} = \alpha \mathbf{B}$ may need to be replaced by a convolution, i.e. a product in wavenumber space. This is rather speculative, but there are indeed some hints from hydromagnetic turbulence simulations that the contributions to α from high wave numbers are diminished (Brandenburg & Sokoloff 1999). The same conclusion is reached by looking at a simple turbulence simulation of helically forced hydromagnetic turbulence, which is discussed now.

In our simple cartesian model of size $(2\pi)^3$ the forcing is adopted in a narrow band of wavenumbers around 10; see Fig. 6. As time goes on the power at progressively larger scales begins to increase. The second panel shows the evolution

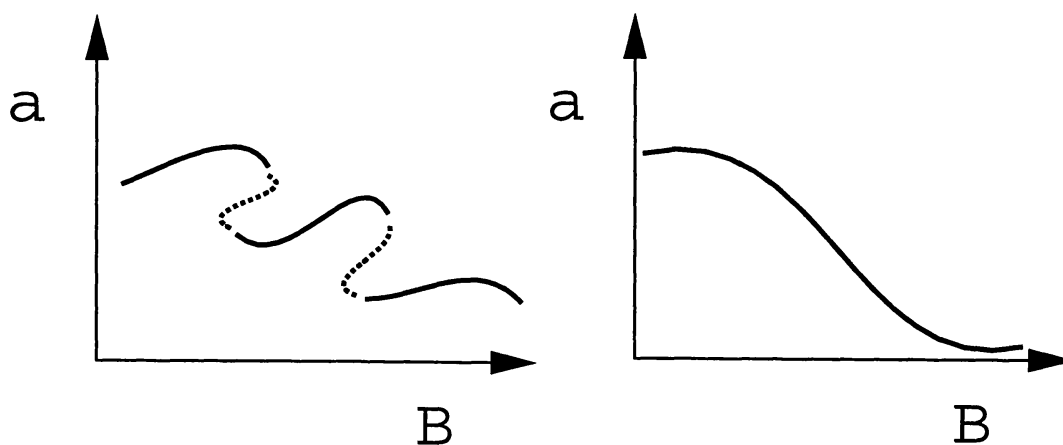


Figure 5. Sketch of α anti-quenching as suggested from interpretations of observations (left) compared with conventional α -quenching (right).

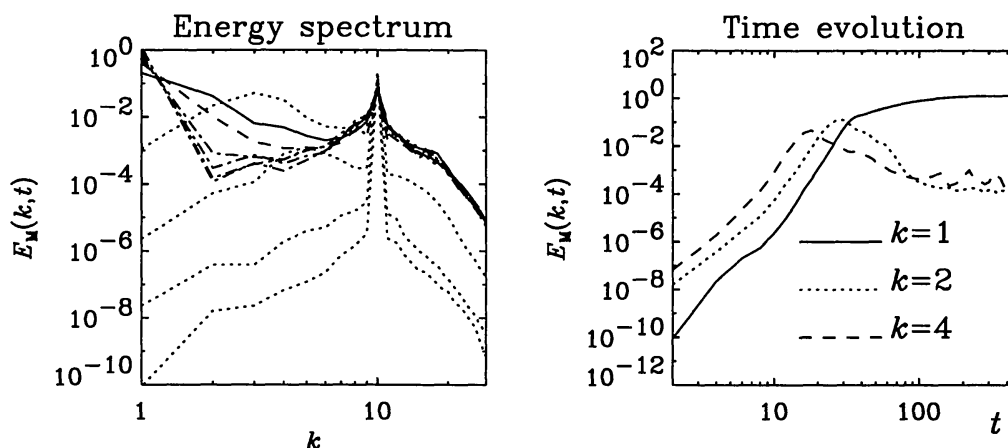


Figure 6. Spectral magnetic energy, $E_M(k, t)$, as a function of wavenumber k for different times: dotted lines are for early times ($t = 2, 4, 10, 20$), the solid and dashed lines are for $t = 40$ and 60 , respectively, and the dotted-dashed lines are for later times ($t = 80, 100, 200, 400$).

of the power at $k = 1, 2$, and 4 . Note that the $k = 4$ mode grows exponentially until it reaches a maximum, after which it begins to fall off again and then settles at a value significantly smaller than the maximum value. This happens once the power at $k = 2$ has saturated. Again, however, once $k = 2$ has reached a maximum the power in that mode diminishes at the expense of the $k = 1$ mode, corresponding to a wave with the largest possible wavelength for a given box size. (Details of this work can be found in Brandenburg (1999) and Bigazzi (1999).

The main conclusion to be drawn from this model is that the large scale field (at $k = 1$) grows until it saturates, and then it begins to suppress the power at

all higher wavenumbers (smaller scales). This is in contrast to recent suggestions that the small scale fields may suppress the growth of the large scale field (Vainshtein & Cattaneo 1992).

3. Magnetically driven α -effects

The other important question that needs to be clarified concerns the suggested increase of the dynamo effect with field strength. In the analysis of Brandenburg, Saar, & Turpin (1998) this was just a hypothesis that appeared plausible in view of other simulated dynamos that share the property of becoming more and more effective as the magnetic field strength increases. One possible and straightforward explanation would be that α may *not* be driven by thermal buoyancy, as in Figs. 1 and 2, but by *magnetic* buoyancy. This idea goes back to Schmitt (1985) who was the first to derive in detail the α -effect resulting from such a system. Recent simulations have been presented by Brandenburg & Schmitt (1998), and model calculations have been carried out by Moss, Shukurov, & Sokoloff (1999). The stronger the magnetic field, the more the flux tubes are evacuated (total pressure = magnetic pressure + gas pressure) and the more buoyant they are. It may therefore not be so implausible that α could indeed increase with increasing field strength.

If α really does increase with field strength we need some other mechanism for saturation of the dynamo. This could be again magnetic buoyancy: once the magnetic buoyancy effect exceeds a certain value it would no longer lead to field growth, because the generated flux would be removed too quickly from the dynamo-active region. In the case of the magnetorotational instability, which is primarily relevant to *accretion discs*, the growth would cease once the Alfvén speed becomes so large that the travel distance of an Alfvén wave within one orbit becomes comparable to some relevant scale of the disc (e.g. the disc height in the case of a vertical field). This would limit effectively the mean field strength. This system provides an important example of a magnetically driven α -effect (Brandenburg et al. 1995, Brandenburg & Donner 1997). Here the turbulence is driven by the magnetorotational or Balbus-Hawley (1991) instability.

Before we move on to discussing further implications of the observed cycle frequencies let us take a closer look at some numerical simulations of hydromagnetic turbulence exhibiting dynamo action.

4. Simulations

Brandenburg, Nordlund, & Stein (1999) have simulated a convective dynamo with imposed shear trying to capture both the effects of latitudinal differential rotation in the convection zone proper and vertical shear at the bottom of the convection zone. In that simulation the total magnetic energy, $\langle \mathbf{B}^2 \rangle$, as well as the energy in the mean magnetic field, $\langle \mathbf{B} \rangle^2$, increase exponentially until saturation is reached. The mean field shows unsteady behavior without real cycles and field reversals. However this is strongly related to geometrical effects and boundary conditions, because the large scale field extends over the scale of the box making global effects important. Furthermore, the energy of the mean field to the total magnetic energy, $f = \langle \mathbf{B} \rangle^2 / \langle \mathbf{B}^2 \rangle$, which is a measure of the

filling factor, also increase with time. Thus, again, the large scale field becomes better defined (relative to the fluctuations) once it reaches appreciable field strength. Those results are encouraging in that they confirm the observational finding that the solar magnetic field shows a great deal of coherence even though it is basically of turbulent origin.

In the case of local turbulence simulations of accretion disc dynamos (Brandenburg et al. 1995) we found that the mean magnetic field (averaged azimuthally and over some radius interval) shows spatio-temporal coherence as evidenced by a “butterfly-type” diagram of the mean toroidal field as a function of time and height above and below the midplane of the disc. This result is however markedly dependent on boundary conditions. If one adopts perfectly conducting boundary conditions instead of vacuum boundary conditions one finds steady dipole-type solutions instead of oscillatory quadrupole-type solutions (Brandenburg 1998b). It is remarkable that the same change of behavior is reproduced by an $\alpha - \Omega$ dynamo model. In that sense simulations and $\alpha - \Omega$ model show an important similarity.

There is another point that needs to be emphasized. While simulations such as the accretion disc simulations show fairly well-defined large scale fields, they also display an extremely “noisy” behavior for the turbulent electromotive force and hence the α -effect. Although it has been shown that in the presence of shear and turbulent diffusion, noisy α -effects are quite capable of producing mean fields that are not very noisy (Vishniac & Brandenburg 1997), it remains still somewhat of a mystery as to how such a noisy α -effect can have anything to do with a fairly well-behaved large scale magnetic field as seen in the simulations.

5. Seat of the dynamo

There have been long-standing speculations whether or not the solar dynamo operates in the overshoot layer or in the convection zone proper. Already the early analysis of Nordlund et al. (1992) showed that the dynamo can operate in the convection zone proper, but that there is a strong downward pumping effect (as seen for example in 3-dimensional animations), that transport the field from the convection zone proper to the bottom of the convection zone. This effect is related to turbulent entrainment of magnetic field in strong concentrated downward plumes. Those downward plumes are a direct consequence of strong density stratification that causes downward flows to converge and upward flows to diverge (e.g., Stein & Nordlund 1989).

The same downward transport for flux would happen as the convection zone becomes deeper. For a fully convective star we would therefore expect an accumulation of flux at the centre, as sketched in Fig. 7.

In the framework of this picture we would thus expect a *smooth* change of behavior from stars with outer convection zones to fully convective stars, as it is seen in the X-ray emission of late-type stars (e.g., Fleming et al. 1995). Note also that in a fully convective star the radial gravity force vanishes at the center. This too helps making the center of the star an obvious place for the magnetic field to stay for sufficiently long times.

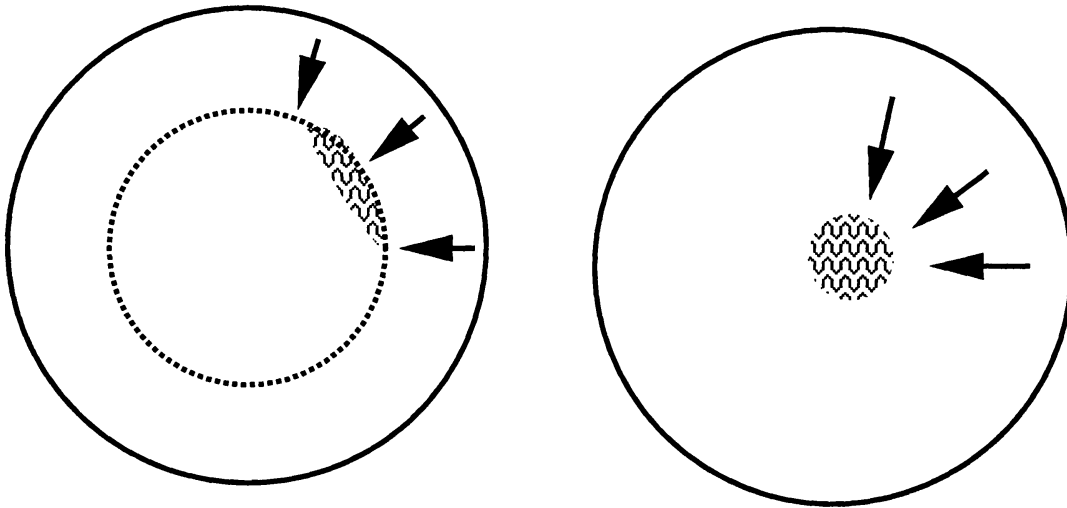


Figure 7. Dynamos with turbulent downward pumping in convective shells and fully convective stars. Magnetic fields are generated in the convection zone proper and then pumped to the bottom of the convection zone, or to the center in the case of a fully convective star.

6. A comment on plotting nondimensional data

Since the paper of Noyes et al. (1984) we know that there is a strong correlation between $\langle R'_{\text{HK}} \rangle$ and the inverse Rossby number, $\text{Ro}^{-1} = 2\Omega\tau_{\text{turnover}}$, where τ_{turnover} is the turnover time of the star. For main sequence stars τ_{turnover} is mainly a function of stellar mass or color, $B - V$. The relation between $\langle R'_{\text{HK}} \rangle$ and Ro^{-1} is valid for both active and inactive stars. For a subset of ~ 20 cyclic stars of the sample of Baliunas et al. (1995), Brandenburg, Saar, & Turpin (1998) found that $\langle R'_{\text{HK}} \rangle \sim \text{Ro}^{-\mu}$ with $\mu \approx 0.99$. The two lines in Fig. 4 correspond then to $\omega_{\text{cyc}}/\Omega \sim \text{Ro}^{-\sigma}$, where $\sigma_I = 0.46$ for the inactive branch and $\sigma_A = 0.48$ for the active branch.

One might think that a plot of $\omega_{\text{cyc}}/\Omega$ versus Ω would be at least approximately similar to a plot of $\omega_{\text{cyc}}/\Omega$ versus $\text{Ro}^{-1} = 2\Omega\tau_{\text{turnover}}$. This is however not the case! Baliunas et al. (1996) found that $\omega_{\text{cyc}}/\Omega \sim \Omega^{\bar{\sigma}}$, where $\bar{\sigma} \approx -1.5$ for both branches together, i.e. without distinguishing between active and inactive branches. This is possible because the omission of the τ_{turnover} factor causes the data on the abscissa to be rearranged such that ω_{cyc} is now almost independent of Ω . Using an extended data set of cyclic stars by including data from certain photometric variables and secondaries of cataclysmic variables Saar & Brandenburg (1999) found that $\omega_{\text{cyc}} \sim \Omega^{-0.2}$, and so $\omega_{\text{cyc}}/\Omega \sim \Omega^{-1.2}$. Nevertheless, the $\omega_{\text{cyc}}/\Omega$ versus $\text{Ro}^{-1} = 2\Omega\tau_{\text{turnover}}$ relation shows a clear separation into inactive, active, and now, for the extended dataset, even superactive stars. For superactive stars the exponent σ_S is *negative*, but small enough that (weak) anti-quenching is still present (Fig. 5).

7. Conclusions

While dynamo theory in its present form is in principle able to reproduce basic behavior of solar and stellar magnetic fields and cycles, there are a number of problems of theoretical and practical nature, as well as a number of new hypotheses that could resolve some of these problems.

The main theoretical problem is related to the functional dependence between the electromotive force and the mean magnetic field. Comparison with simulations suggests tentatively that α may work preferentially at the largest possible scale. If that is true one could solve the (practical) problem of explaining the increase of stellar cycle frequencies with increasing inverse Rossby number by assuming that the α -effect increases with field strength (anti-quenching).

Another rather practical problem concerns the shape of the solar butterfly diagram. Theoretically one would expect that the dynamo wave should migrate poleward; see the pioneering simulations of Gilman (1983) and Glatzmaier (1985). In order to explain the observed equatorward migration one would either need to have a negative α in the northern hemisphere (some simulations do predict this, but it is not clear that this applies really to the solar regime), or one might be able to explain the migration directly by invoking a suitable meridional circulation. Recent work by G. Rüdiger and collaborators (private communication) suggests that this is indeed a viable possibility (see also Dikpati & Charbonneau 1999). This was first suggested by Parker (1987) and Durney (1996), and confirmed by a model calculation by Choudhuri et al. (1995), but it seemed to be a rather special case given that meridional circulation usually tends to make oscillatory models stationary (Rädler 1986).

In any case, the theoretical foundations of $\alpha - \Omega$ are sufficiently shaky that one may consider a realistic high-resolution simulation of stellar dynamos as absolutely crucial before one can try to use $\alpha - \Omega$ type dynamos with real predictive power.

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