Solar-stellar dynamos as revealed by helio- and asteroseismology
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From fibril to diffuse fields during dynamo saturation

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Abstract. The degree of intermittency of the magnetic field of a large-scale dynamo is considered. Based on simulations it is argued that there is a tendency for the field to become more diffuse and non-intermittent as the dynamo saturates. The simulations are idealized in that the turbulence is strongly helical and shear is strong, so the tendency for the field to become more diffuse is somewhat exaggerated. Earlier results concerning the effects of magnetic buoyancy are discussed. It is emphasized that the resulting magnetic buoyancy is weak compared with the stronger effects of simultaneous downward pumping. These findings are used to support the notion that the solar dynamo might operate in a distributed fashion where the near-surface shear layer could play an important role.

1. Introduction

In the early days of dynamo theory the degree of intermittency of the generated magnetic field was not much of an issue. However, with the development of mean-field theory it became clear that the magnetic field can be thought of as consisting of a mean component together with a fluctuating one. The fluctuating component was initially thought to be weak, but that too changed when it was realized that at large magnetic Reynolds numbers the fluctuations can strongly exceed the level of the mean field.

Given the intermittent nature of solar magnetograms, the surface magnetic field can well be described as fibril. This description was introduced by Parker (1982) to emphasize that such a field may have rather different properties than a more diffuse field. The fibril nature of the magnetic field is particularly well illustrated by the fact that sunspots are relatively isolated features covering only a small fraction of the solar surface. It is often assumed that the fibril magnetic field structure extends also into deeper layers. On the other hand, observations of sunspots suggest that spots are rather shallow phenomena (Kosovichev 2002). Furthermore, simulations of turbulent dynamos tend to show that the dynamo-generated magnetic field becomes less fibril as the fraction of the mean to the total magnetic field increases. Such dynamos are generally referred to as large-scale dynamos (as opposed to small-scale dynamos) and they require either kinetic helicity or otherwise some kind of anisotropy. These ingredients are generally assumed to be present in the Sun, and they are also vital for many types of mean-field dynamos, in particular the $\alpha\Omega$-type dynamos. It is therefore of interest to study in more detail the dependence of the degree of intermittency of the field on model parameters.

The significance of looking at the degree of intermittency of the Sun's magnetic field is connected with the question of how important is magnetic buoyancy in transporting mean magnetic field upward to the surface and out of the Sun (Moreno-Insertis 1983). Magnetic buoyancy may therefore act as a possible saturation mechanism of the dynamo (see, e.g., Noyes et al. 1984), with the consequence of nearly completely wiping out magnetic fields of equipartition strength within the convection zone. On the other hand, if magnetic buoyancy is not a dominant effect, the dynamo may operate in a much more distributed fashion (Brandenburg 2005).

2. Fully helical dynamos

Let us begin by looking at an idealized case of a dynamo in the presence of fully helical forcing. We shall distinguish between the kinematic regime where the field is weak and still growing exponentially, and the dynamic regime where the field is strong and beginning to reach saturation field strength. In Fig. 1 we plot the dependence of the mean-squared values of the small-scale and large-scale fields defined here by horizontal averages, so $\boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{b}$, where $\overline{\boldsymbol{B}}$ and $\boldsymbol{b}$ have been defined as the mean and fluctuating fields. Note that in the kinematic regime the energy of the magnetic fluctuations exceeds that of the mean field by a factor of about 3, while in the dynamic regime this ratio is only about 1/3. Here we have used data from a recent paper of Brandenburg (2009) were the magnetic Reynolds number is only about 6, while the fluid Reynolds number is 150, so the magnetic Prandtl number is 0.04. The turbulence is forced with a maximally helical forcing function at a wavenumber of about 4 times the minimal wavenumber of the domain. This ratio is also called the scale separation ratio and it also determines the ratio of magnetic fluctuations to the mean field in the kinematic regime, and its inverse in the dynamic regime (Blackman & Brandenburg 2002).

Depending on the value of the magnetic Prandtl number Pr_M, i.e. the ratio of kinematic viscosity to magnetic diffusivity, the field can be rather intermittent and lack large-scale order, especially when the magnetic Prandtl number is not small; see Fig. 2. Note however the emergence of a large-scale pattern in the kinematic stage for $\mathrm{Pr}_M = 0.01$, while for $\mathrm{Pr}_M = 0.1$ there are only a few extended patches and for $\mathrm{Pr}_M = 1$ the field is completely random and of small scale only. However, when the dynamo saturates, a large-scale structure emerges regardless of the value of the magnetic Prandtl number and the field is considerably less intermittent than in the early kinematic stages. These simulations (Brandenburg 2009) were used to argue that in the Sun, where Pr_M is very small, the onset of large-scale dynamo action should not depend on the actual value of Pr_M, even though the onset of small-scale dynamo action does depend on it (Schekochihin et al. 2005; Iskakov et al. 2007).

Another example is forced turbulence in the presence of a systematic shear flow that resembles that in low latitudes of the solar convection zone and open boundary conditions at the surface and the equator. Such a model was studied by Brandenburg & Sandin (2004) to determine how the α effect is modified in the presence of magnetic helicity fluxes, and by Brandenburg (2005) in order to determine the structure of dynamo-generated magnetic fields. In Fig. 3 we

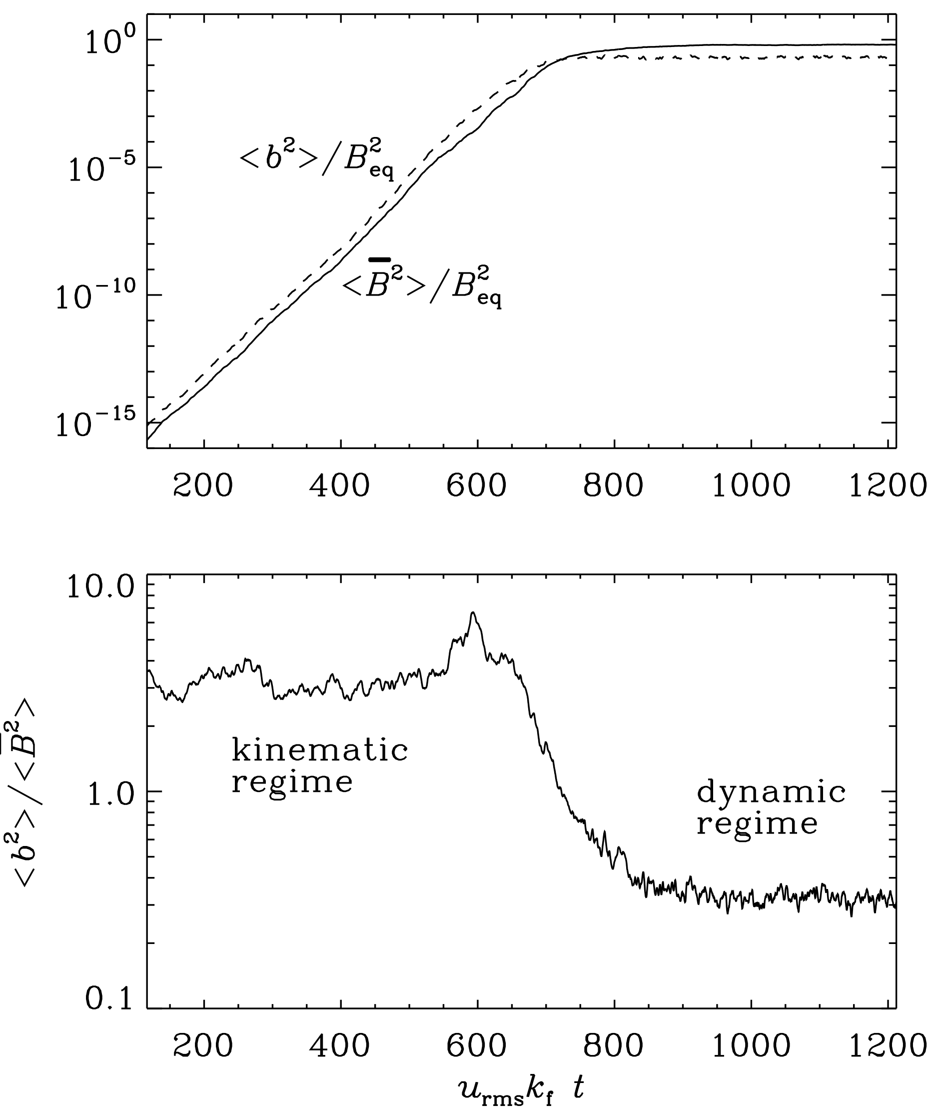

Figure 1. Dependence of the normalized mean-squared value of the mean field $\langle\overline{\boldsymbol{B}}^2\rangle$ (solid line in the upper panel) and the fluctuating field $\langle\boldsymbol{b}^2\rangle$ (dashed line in the upper panel) and their ratio (lower panel) for $\mathrm{Re}_M = 6$ and $\mathrm{Pr}_M = 0.04$. The equipartition field strength $B_{\rm eq} = \langle\mu_0\rho\boldsymbol{u}^2\rangle$ has been introduced for normalization purposes.

compare meridional cross-sections of the toroidal component of the magnetic field at a kinematic time ($tu_{\rm rms}k_{\rm f} = 100$) with that at a later time when the dynamo has saturated and a large-scale field has developed ($tu_{\rm rms}k_{\rm f} = 1000$), where $u_{\rm rms}$ is the turbulent rms velocity and $k_{\rm f}$ is the wavenumber of the energy-carrying eddies or the forcing wavenumber in this case.

The case shown in Fig. 3 looks like the magnetic Reynolds number is small, but this is not really the case. In fact, the magnetic Reynolds number based

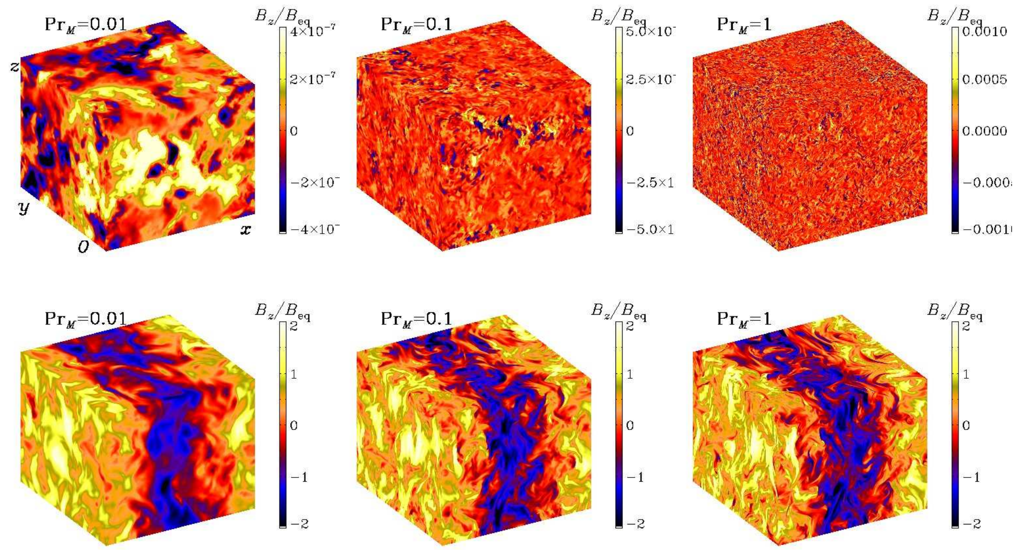

Figure 2. Visualization of one component of the magnetic field in the kinematic regime (upper row) compared with the saturated regime (lower row) for magnetic Prandtl numbers ranging from $\mathrm{Pr}_M = 0.01$ to 1 at $\mathrm{Re} = 670$. The orientation of the axes is indicated for the first panel, and is the same for all other panels. Adapted from Brandenburg (2009).

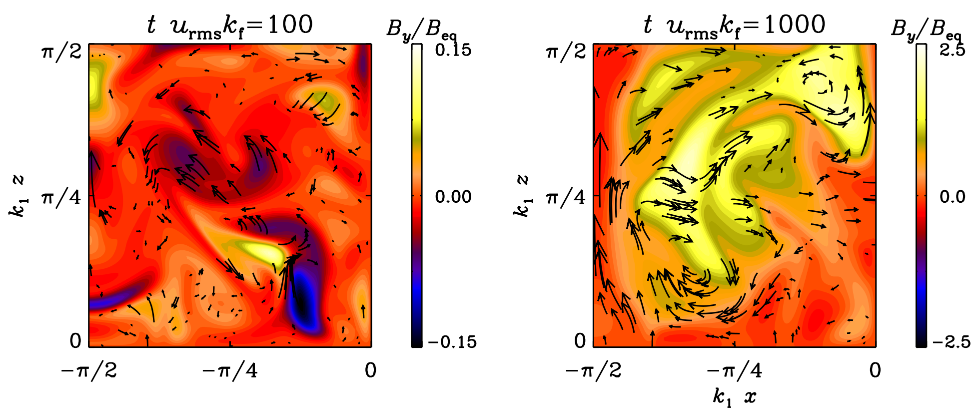

Figure 3. Snapshots of the magnetic field in the meridional plane during the kinematic stage ($t = 100$ turnover times, left panel) and the saturated stage ($t = 1000$ turnover times, right panel). Vectors in the meridional plane are superimposed on a color/gray scale representation of the azimuthal field. The color/gray scale is symmetric about red/mid-gray shades, so the absence of blue/dark shades (right panel) indicates the absence of negative values. Note the development of larger scale structures during the saturated stage with basically unidirectional toroidal field. Adapted from Brandenburg (2005).

on the inverse wavenumber of the energy-carrying eddies, $u_{\rm rms}/\eta k_{\rm f}$, is about 80. Here, $k_{\rm f}/k_1 = 5$ is the forcing wavenumber in units of the smallest wavenumber in the domain, $k_1 = 2\pi/L$, where L is the toroidal extent of the computational domain. So, the magnetic Reynolds number based on L, which is sometimes also quoted, would then be about $2\pi \times 5 \approx 30$ times larger, i.e. about 2400.

Note also that, unlike the early kinematic stage when there can still be many sign reversals, at later times the field points mostly in the same direction. Indeed, the toroidally averaged magnetic field captures about 50% to 70% of the total magnetic energy in the saturated state.

These simulations confirm that there is a clear tendency for the magnetic field to become less intermittent and more space-filling and diffuse as the dynamo saturates. It must be noted, however, that these simulations are idealized in that the turbulence is driven by a forcing function that is maximally helical, and that the shear is relatively strong, i.e. the shear-flow amplitude is about five times stronger than the rms velocity of the turbulence. In the Sun this ratio is about unity. Therefore one must expect that the degree to what extent the field tends to become more diffuse is in reality less strong than what is indicated by the simulations presented here.

3. Magnetic buoyancy

In the early 1980s, dynamo theory was confronted with the issue of magnetic buoyancy (Spiegel & Weiss 1980). It was thought that buoyant flux losses would reduce the dynamo efficiency. This effect was then also built into dynamo models of various types as a possible saturation mechanism (Noyes et al. 1984, Jones et al. 1985, Moss et al. 1990). However, with the first compressible simulations of turbulent dynamo action (Nordlund et al. 1992) it became clear that magnetic buoyancy is subdominant compared with the much stronger effect of turbulent downward pumping. Figure 4 shows a snapshot from a video animation of magnetic field vectors together with those of vorticity (Brandenburg & Tuominen 1991). The magnetic field forms flux tubes that get wound up around a tornado-like vortex in the middle.

In Fig. 5 this magnetic buoyancy of the flux tubes is analyzed in more detail. This figure confirms that there is indeed magnetic buoyancy, but it is balanced in part by the effects of downward pumping and the explicit downward motion in the proximity of the downdraft where the field is most strongly amplified during its descent.

4. Connection with distributed dynamos

We have discussed the nature of the magnetic field of a large-scale dynamo in the saturated regime and have argued that the field becomes diffuse and more nearly space-filling as the dynamo saturates and that the effects of magnetic buoyancy are weak compared with the downward motions associated with the strong downdrafts in convection. Here we have mostly focused on earlier simulations, but it is important to realize that at the moment there is no general agreement about the detailed nature of the solar dynamo. Is it essentially of $\alpha\Omega$ type, or are there other more dominant effects responsible for generating a large-scale magnetic field? What causes the equatorward migration of the

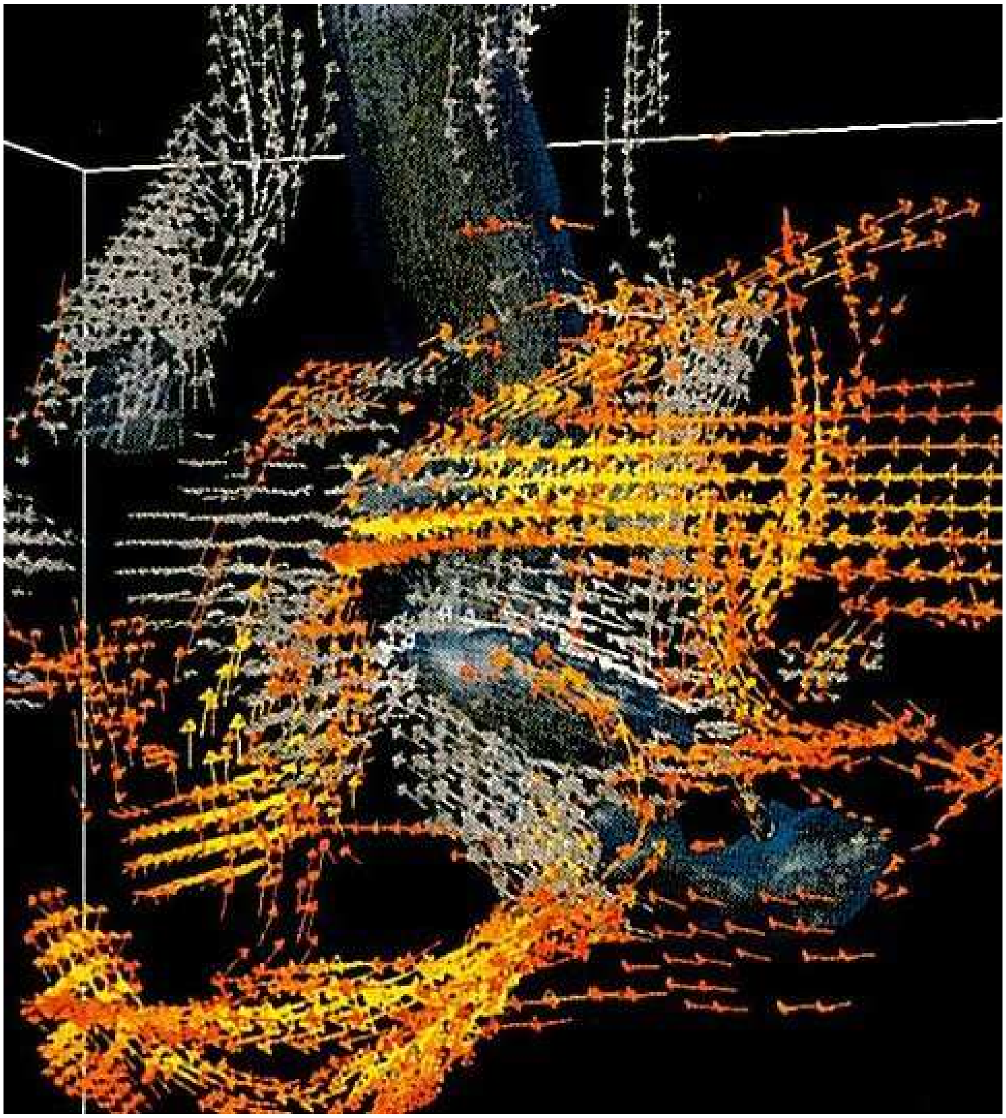

Figure 4. Snapshot from a video animation showing magnetic field vectors in yellow (the strongest) and orange (less strong) together with those of vorticity in white. Transparent surfaces of constant negative pressure fluctuations are shown in blue. Note that the vectors of magnetic field form flux tubes that get wound up around a tornado-like vortex in the middle. Adapted from Brandenburg & Tuominen (1991).

toroidal magnetic flux belts? Is it the dynamo wave associated with the $\alpha\Omega$ dynamo, or is it the meridional circulation that overturns the intrinsic migration direction (Choudhuri et al. 1995; Dikpati & Charbonneau 1995). What is the dominant shear-layer in the Sun for the $\alpha\Omega$ dynamo to work? There is first of all latitudinal shear, which is the strongest in absolute terms, and important for amplifying toroidal magnetic field as well as promoting cyclic dynamo action (Guerrero & de Gouveia Dal Pino 2007). In addition, there is radial shear which

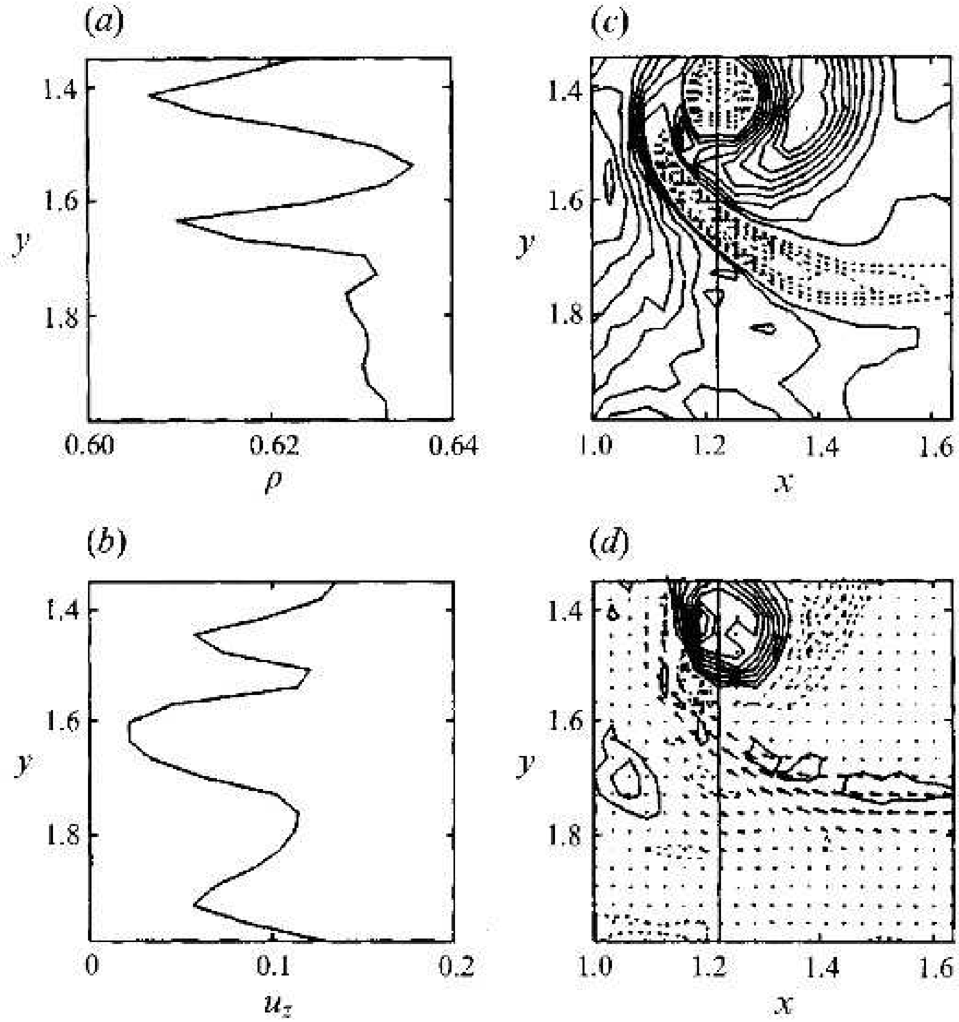

Figure 5. Panel (a) shows a horizontal density profile through the line indicated in panel (c) by the line in the corresponding horizontal cross-section through a strong magnetic flux tube. Dotted contours indicate values less than the average. Panel (b) gives the vertical velocity (positive values mean upward motion) along the line indicated in the horizontal cross-section in panel (d). Here the vectors indicate magnetic field vectors. Adapted from Brandenburg et al. (1996).

might be important for determining the migration direction of the toroidal flux belts. However, it is not clear whether the relevant component here is the positive $\partial\Omega/\partial r$ in the bulk or the bottom of the convection zone, or the negative $\partial\Omega/\partial r$ at the bottom of the convection zone at higher latitudes. Or is it the negative $\partial\Omega/\partial r$ in the near-surface shear layer?

An attractive property of the latter proposal is that it would allow for a dynamo scenario that is in many respects similar to that envisaged in the early years of mean-field dynamo theory (Steenbeck & Krause 1969, Köhler

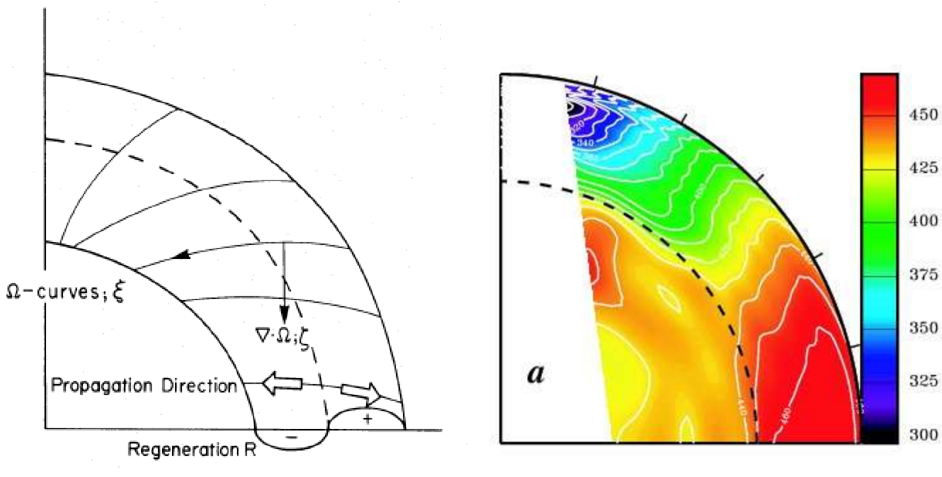

Figure 6. Comparison of the differential rotation contours that were originally expected by Yoshimura (1975) based on solar dynamo model considerations (left) with those by Thompson et al. (2003) using helioseismology (right). Note the similarities between the contours on the left (over the bulk of the convection zone) and those on the right (over the outer 5% of the solar radius).

1973, Yoshimura 1975). In Fig. 6 we show the structure of Ω contours as they were estimated by Yoshimura (1975) based on the constraint that the internal angular velocity matches the latitudinal differential rotation at the surface and that $\partial\Omega/\partial r$ is negative in the interior so that the dynamo wave propagates equatorward. The relative strength of the negative Ω gradient near the surface is truly amazing and is best seen in a plot of Benevolenskaya et al. (1998), which shows the radial dependence of Ω at different latitudes (Fig. 7). The fact that the radial gradient is so strong is in principle not new. Indeed, a mismatch between the higher helioseismic results for Ω some 40 Mm below the surface and the lower values from Doppler measurements of the photospheric plasma was recognized since the 1980s, but it is only now that helioseismology can actually provide detailed data points nearly all the way to the surface.

We emphasize that these proposals ignore the possibility that the meridional circulation could in principle turn the direction of propagation around and might produce equatorward migration even with a positive $\partial\Omega/\partial r$ (Choudhuri et al. 1995, Dikpati & Charbonneau 1999). However, this requires that the induction effects given by α and the radial differential rotation are separated in space, just as it is the case for the Babcock-Leighton dynamo effect. Although such a hypothesis was already made by Steenbeck & Krause (1969) for other reasons, it is not clear that this is or will be compatible with results of turbulence simulations.

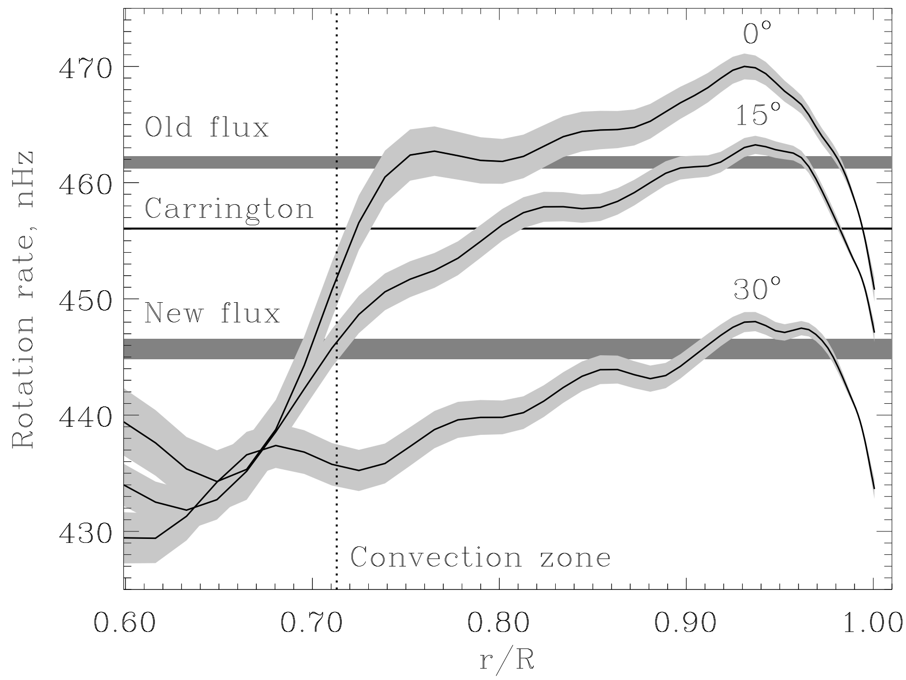

Figure 7. Radial profiles of angular velocity as given by Benevolenskaya et al. (1999). Note the sharp negative radial gradient near the surface.

5. Unexplored effects

There are two important issues that need to be clarified in the context of distributed dynamos. One is connected with the question why the dynamo might work efficiently in the near-surface shear layer in spite of the opposing effects of downward pumping, for example. The other is related to the formation of active regions and sunspots in models lacking strong fields of $\sim 100\,\mathrm{kG}$ strength at the bottom of the convection zone, as is expected based on Joy's law and results from the thin flux tube approximation (Chou & Fisher 1989; Choudhuri & D'Silva 1990).

Regarding the first issue one might expect that it could be connected with magnetic helicity conservation, which is now recognized as a major culprit in causing so-called catastrophic quenching of large-scale dynamo effects (see Brandenburg & Subramanian 2005 for a review). Alleviating such catastrophic quenching is facilitated by magnetic helicity fluxes connected with scales that are shorter than those of the large-scale field of the 11-year cycle. Disposing of such excess magnetic helicity should be easier near the surface than deeper down, making the near-surface shear layer more preferred for dynamo action. Regarding the formation of active regions and sunspots, some important clues have been obtained by investigating mean-field turbulence effects both in the momentum and in the energy equations. We refer here to the work of Kitchatinov & Mazur (2000) who find that a self-concentration of magnetic flux is possible as a result of the magnetic suppression of the turbulent heat flux. Another mechanism might be connected with negative turbulent magnetic pressure effects; see Rogachevskii & Kleeorin (2007) for a recent reference on this subject. Clarifying these questions would be critical before further pursuing the idea of distributed dynamo action in the Sun.

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