OBSERVATIONAL CONSTRAINTS FOR SOLAR-TYPE DYNAMOS

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ABSTRACT: The different phenomena of solar and stellar activity are generally considered to have its origin in the turbulent convective envelopes of these stars. We will discuss how the problem can be treated in the framework of the mean-field concept.

The solar differential rotation can be produced by Reynolds stresses parameterizing the influence of rotation on convection. Some forms of these stresses give the observed solar differential rotation and other related observed phenomena, including the internal angular velocity in the convection zone, obtained by helioseismology.

As a further problem we investigate whether it is possible to produce the observed geometry of the mean oscillating magnetic field of the Sun when taking into account the constraints given by differential rotation and mean helicity of convection, and furthermore, whether this magnetic field produces correctly the observed cyclic flows, e.g. the so-called torsional oscillations.

Some solutions of these problems will be summarized. With simplified examples it is further discussed how the procedure can be extended to different types of convection zones, in order to derive observable properties of other stars with convective envelopes.

1. THE SOLAR CYCLE

The theory of the oscillating dynamo formulated about 20 years ago considered only the solar butterfly diagrams, i.e. the solar activity cycle represented by the behaviour of the toroidal magnetic fields. Concerning the poloidal field, only its smallness in comparison with the toroidal field was known. It is not an easy task to observe weak magnetic fields and to define the mean poloidal field at the solar surface. At present the interplay of the solar field components is known with some certainty. For example, now we know that the polar fields reversed polarity in 1957/58 and in 1969/1971 and 1980, i.e. very close to the activity maximum.

Most likely we also know the process of the reversals of the poloidal fields. The new polarity appears 2-3 years before the maximum at the heliographic latitude about 30° and moves its upper limit slowly towards the poles arriving there at the time of maximum activity. From this time on, the new polarity of the polar field prevails (e.g. Leroy and Noens, 1983; Hoeksema, 1984; Makarov and Makarova, 1986). In theoretical models this is described by a second, poleward moving branch of the field belts (Stix, 1974; Yoshimura, 1976).

We have also information about the phase constellation with respect to the sign of the field components. Almost during the entire cycle, poloidal and toroidal field are out of phase and have opposite signs: $\bar{B}_r \bar{B}_\phi < 0$ (Stix, 1983). This property has immediate consequences for the longitudinal Lorentz force,

$$L_\varphi \approx \frac{\bar{B}_r}{\mu} \frac{\partial \bar{B}_\varphi}{\partial r} \approx -\frac{\bar{B}_r \bar{B}_\varphi}{\mu (R-r)} \tag{1.1}$$

O. Havnes et al. (eds.), Activity in Cool Star Envelopes, 13–20.

(for $r < R$), and determines the magnetically driven longitudinal flows. Because of the negative sign of $\bar{B}_r\bar{B}_\phi$, they should flow in the direction of the basic rotation (at least where $\bar{B}_r\bar{B}_\phi$ has its maximum), increasing the local angular velocity.

Recently such flows have been observed in detail. Their inventors called them "torsional oscillations" (Howard and LaBonte, 1980) and they are part of the 11-year cyclic variation of the solar rotation law. Possibly we have now a better chance to design the solar dynamo in order to analyze the solar interior. Most likely the lack of the dynamo theory in the past was simply a deficit of observations, which did not fix sufficiently many parameters.

These magnetically driven flows can be classified by means of the series expansion

$$\Omega\,(\theta,t\,) = \sum_n \Omega_n\;(\,t\,)\;P_n^1\,(\cos\theta)\,/\,\sin\theta \tag{1.2}$$

in which all coefficients have cyclic behaviour. This basic result has been established by spectroscopic means (LaBonte and Howard, 1982; Snodgrass and Howard, 1985) as well as by studying motions of classical tracers (Tuominen et al., 1983; Tuominen and Virtanen, 1984). The variation of the modes $n = 1$, 3 and 5 form the time-dependence of the rotation law, while the higher order terms are the original "torsional oscillations" (Howard and LaBonte, 1980). The latter represent the observation that there exist two zones in both hemispheres with locally enhanced rotation and two zones with slower motion. The whole pattern moves in 22 years from the poles to the equator. The preceding one of the faster belts is always situated between the solar activity zone (the sunspot belt) and the equator.

Quite similar behaviour also holds for the meridional circulation observed at the solar surface. The main exception is that the time-independent part of the velocity seems to be small (< 10 m/s). Again the activity zone divides the flow into two parts, the poleward part being directed polewards and vice versa (Tuominen et al., 1983, Tuominen and Virtanen, 1984, 1987):

$$\bar{u}_{\theta\,cyc}\;\bar{u}_{\varphi\,cyc} > 0 \tag{1.3}$$

The observations reveal the low-mode coefficients Ω_3 and Ω_5 to vary in phase. They have a maximum at the activity maximum and minimum at the activity minimum. At the minimum the rotation law is flattened near the equator and steepened near the poles. Moreover, Gilman and Howard (1984) obtained from MtWilson sunspot data that the sunspot belt rotates faster at the activity minimum, while Tuominen and Virtanen (1987) showed on the basis of the Greenwich data that this effect follows basically from the change of the profile of the differential rotation and that the equatorial velocity in itself has a maximum at the activity maximum.

In our interpretation of those phenomena as a flow system driven by Lorentz force (Rüdiger et al., 1986) the migration feature of the pattern plays a basic role. In order to obtain the observed migration period of 22 years, the Lorentz force component $\bar{B}_r\bar{B}_\phi$ (which is the angular momentum transport by magnetic field) is only allowed to form two main belts travelling 45° in 11 years. This is realized for dynamo models with a deep convection zone. Probably the depth of the convection zone is the basic parameter which determines the number of the Lorentz force belts, in latitude and below the surface. If this finding is true, the observations of torsional oscillations in connection with the other cyclic variations would provide an interesting tool for studying outer stellar convection zones.

Today a dynamo model must reproduce not only the butterfly diagram and the phase constellation of both field components but also the Lorentz force driven flows. Fortunately the new observational and theoretical findings concerning the rotation law below the solar surface make the difficult problem somewhat easier (Duvall et al., 1984; Hill, 1987; Rüdiger and Tuominen, 1987; Durney, 1987).

We have studied a dynamo model having a rotation law of the solar convection zone as it is summarized by Hill (1987): a small maximum of angular velocity very close to the surface, a slow decrease of 3 % to the middle of the convection zone and an increase of 10 % to the bottom of the convection zone. Furthermore, the latitudinal differential rotation remains the same down to the bottom of the convection zone as observed at the surface, i.e. 30 % difference between the equator and the polar axis. This picture should be considered tentative (see e.g. Brown and Morrow, 1987).

There is no difficulty in producing the main properties of an oscillating magnetic field with these constraints. The effect of the latitudinal differential rotation is mainly to create a poleward branch (see also Yoshimura, 1975), but at the same time it raises the position of the maximum toroidal field too high into mid-latitudes. The inclusion of a higher order term in the expansion of α (Rüdiger, 1980; Schmitt, 1987):

$$\alpha = \alpha_1(r)\ P_1(\cos\theta)\ +\ \alpha_3(r)\,P_3(\cos\theta)\ +\ \ldots \tag{1.4}$$

leads to the observed field geometry and phase relations, if we take $\alpha_3 \approx -\alpha_1$. This form of α resembles the one used by Yoshimura (1975). It prefers, however, slightly the quadrupolar parity and violates the important constraint of Hale's polarity law. This law demands the preference of dipolar parity. This means that the marginal dynamo number of the dipolar mode should be somewhat smaller than the marginal dynamo number of quadrupolar parity. In this case the only stable non-linear solution will be the one with odd parity as suggested by Krause and Meinel (1987).

A great number of other non-linear problems remains to be solved. These include the field strengths, the interaction of various modes, which may have connexion with the result obtained by Stenflo and Vogel (1986) concerning the dispersion relation for the even parity modes of the field, the amplitude - period relation (Waldmeier, 1935), and the secular variations (e.g. Maunder and other Grand minima). Also the sectorial structure in presence of differential rotation of the magnetic field remains an open question (Rädler, 1986a).

2. STELLAR CYCLES

Investigation of stellar cycles supports the study of the Sun as well. The dynamo theory works with electromagnetic induction, produced by flows in rotating turbulent media. Rotation and anisotropy in the turbulence play the basic roles. It is impossible to check the dependences of helicity and differential rotation on Ω (rotation) and g (anisotropy) by studying only one star, the Sun. For this reason we need samples of stars with solar type cycles and different Ω and g. Such a sample did not exist 20 years ago. We had only the old statement by Eberhard and Schwarzschild (1913) that giant stars like Arcturus and Aldebaran exhibit solar-type activity found from the emission cores of the

Ca H and K lines: "It remains to be shown whether the emission lines of the star have a possible variation in intensity analogous to the sunspot period".

Much later O.C. Wilson (1978) observed a sample of solar-type stars during several years and found solar-like cycles in many of them. Belvedere et al. (1980) tried to apply the dynamo theory to such stars and predicted an increasing cycle period towards later type stars, whereas Robinson and Durney (1982) computed models having decreasing periods. The observations, however, do not yet show a clear trend (see Baliunas and Vaughan, 1985).

But nevertheless the rotation rates play an important rôle. It became observable in many cases with the same Ca II method. A first inspection showed the number of rotations per cycle for one and the same spectral type to be rather constant (Noyes et al., 1984). Quantitatively, a relation

$$\Omega_{\mathrm{cyc}} \propto \Omega^{*\,1.25} \tag{2.1}$$

applies, with $\Omega^* = 2\,\Omega\,\tau_{\mathrm{corr}}$ and τ_{corr} being the turnover time of eddies near the bottom of the convection zone (as from mixing length theory). This relation probably has an important consequence for the physics of convection zones. In order to reproduce the 22-year period the old models (Steenbeck and Krause, 1969) worked with separate centres of α-effect and differential rotation. The cycle length followed from the geometry of the convection zone rather than from the dynamo number (i.e. from the rotation period). Models with distributed α and $\partial\Omega/\partial r$ work in a different way (Kleeorin et al., 1983). As a demonstration they may be represented by plane dynamo waves:

$$\dot{A} + k^2 A = 2\,D\,B \qquad \dot{B} + k^2 B = i\,k\,A \tag{2.2}$$

where the dynamo number D represents the product of α and $\partial\Omega/\partial r$. It is, therefore, proportional to Ω^2:

$$D \propto \Omega^2 \tag{2.3}$$

The dispersion relation obtained from (2.2) gives for the frequency Ω_{cyc} of the most unstable mode the approximation

$$\Omega_{\mathrm{cyc}} \propto D^{2/3} \tag{2.4}$$

so that

$$\Omega_{\mathrm{cyc}} \propto \Omega^{4/3} \tag{2.5}$$

This relation is surprisingly close to the observations. If our assumptions are essentially correct, then the stellar observations suggest a rather flat distribution of α and $\partial\Omega/\partial r$ in the stellar convection zones, e.g. in that of the Sun.

Let us now turn to the behaviour of Ω_{cyc} and Ω with respect to the depth of the convection zone. As is well-known (Steenbeck and Krause, 1969) the normalized frequency C' and dynamo number C_1 of a dynamo model are defined as

$$C' = \Omega_{\mathrm{cyc}} R^2/\eta \qquad C_1 = \alpha\,\Delta\Omega\,R^3/\eta^2 \tag{2.6}$$

where $\Delta\Omega$ characterizes the differential rotation and η is the turbulent magnetic diffusivity. With the order of magnitude estimate

$$\Delta\Omega \propto r\,\partial\Omega/\partial r \propto \Omega \tag{2.7}$$

and

$$\alpha \propto l\,\Omega \tag{2.8}$$

(l is the "mixing length") we find

$$\frac{\sqrt{C_1}}{C'} \propto \sqrt{\frac{l}{R}}\,\frac{\Omega}{\Omega_{\mathrm{cyc}}} \tag{2.9}$$

It may make sense to put the mixing length equal to the depth of the convection zone. Hence

$$\frac{\sqrt{C_1/d}}{C'} \propto \frac{\Omega}{\Omega_{\mathrm{cyc}}} \tag{2.10}$$

is obtained, where d is the fractional depth of the convection zone.

So we have the possibility to compare the observed number of rotations per cycle ($\Omega/\Omega_{\mathrm{cyc}}$) with results obtained for oscillatory dynamo models having different convection zones. In Fig.1 we present the observations (cf. Noyes et al., 1984, Baliunas and Vaughan, 1985) and in Fig. 2 the left-hand side expression of (2.10) for the model of Steenbeck and Krause (1969, Table 4) and Rädler (1986b, Fig. 16). In these models the cycle period arises from the time which is necessary for a dynamo wave to travel from the maximal helicity centre to the centre of maximal differential rotation. Comparing the slopes in Fig. 1 and Fig. 2 (left panel), we see that this concept of separated induction layers is not in accordance with the observations.

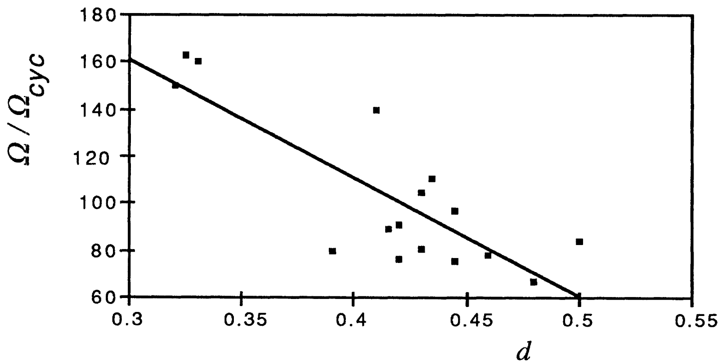

Fig. 1: The observed ratio of angular velocity and cycle frequency versus fractional depth of the convection zone d (the data are taken from Noyes et al., 1984, and Baliunas and Vaughan, 1985).

Let us now compare the observations with models having distributed induction layers. The theory of plane dynamo waves shows that their periods are bounded by an upper value, the Parker period, and that their dependence on the geometry is weaker than in the case of separated induction zones (Kleeorin et al., 1983). We have calculated such models starting from the profiles of Steenbeck and Krause, but with smoother profiles and we have also calculated models with more realistic profiles of differential rotation, described in the previous section (Fig. 2, right panel). The latter models show indeed a rather flat dependence of the number of rotations per cycle on the fractional depth of the convection zone.

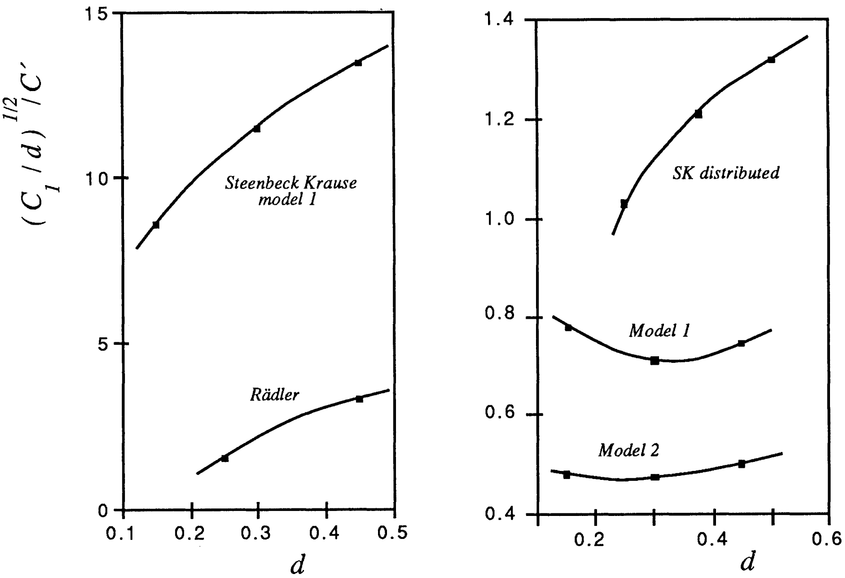

Fig. 2: Left: The quantity on the left-hand side of eq. (2.10) for the model of Steenbeck and Krause (1969, Table 4) and Rädler (1986b, using the same profiles of induction as in his Fig.16). Right: The same but for models with smoother and distributed profiles. "SK distributed" is a model with the error function profiles as used by Steenbeck and Krause, but smoother and overlapping. Models 1 and 2 refer to our model with a differential rotation profile from Hill (1987), taking α and η from the mixing length theory. Model 2 includes the latitudinal differential rotation and α_3.

As a last point we plot in Fig. 3 the dependence of the normalized frequency on the square root of the dynamo number for the same sample of stars, using the estimates (2.7) and (2.8) as before. The slope in Fig.3 is 1.7 and the scatter quite small. The fact that the slope differs from one suggests that the estimates in (2.7) and (2.8) are not necessarily linear in Ω. We can also calculate the same relation for the dynamo models.

Unfortunately the slope does not depend very much on the model: for all cases represented in Fig. 2, this number is between 2.3 and 2.9, in other words larger than suggested by the observations.

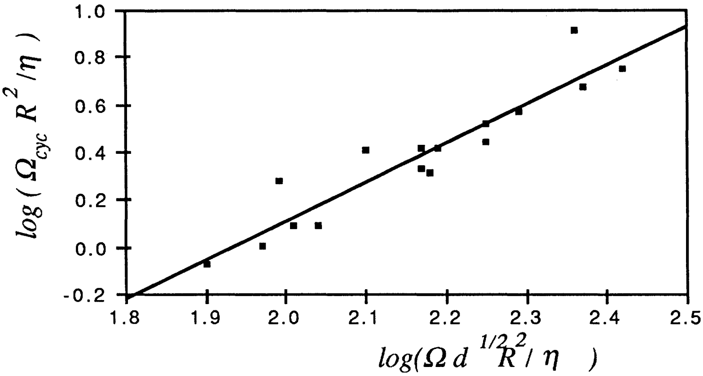

Fig. 3 Plot of $\log(\Omega_{cyc} R^2 / \eta)$ versus $\log(\Omega\ d^{1/2} R^2 / \eta)$, using thesame observed values for Ω_{cyc} and Ω as in Fig.1. The values of *d, R,* and η are taken form Belvedere et al. (1980).

3. FINAL REMARKS

The theory of the solar dynamo can be developed further using all relevant new observations of the magnetic field and associated cyclic flows. The cycles in other solar type stars give further constraints, the most immediate of which is the behaviour of the cycle period. We have discussed the possibility of using the available observations to investigate the distribution of induction effects in stellar convection zones. We found with simple arguments and with calculated dynamo models that the observations do not favour the models with separated layers of induction, i.e. of the α-effect and differential rotation. On the contrary, models with more distributed induction layers in the convection zone reproduce better the observations.

REFERENCES

Baliunas, S.L., Vaughan, A.H.: 1985, *Ann. Rev. Astron. Astrophys.* **23**, 379.

Belvedere, G., Paternò, L., Stix, M.: 1980, *Astron. Astrophys.* **91**, 328.

Brown, T.M., Morrow, C.A.: 1987, *Astrophys. J.* **314**, L21.

Durney, B.R.: 1987, in *The Internal Solar Angular Velocity*, eds. B. R. Durney and S. Sofia, D. Reidel, Dordrecht.

Duvall, T.L.Jr., Dziembowski, W.A., Goode, P.R., Gough, D.O., Harvey, J.W., Leibacher, J.W.: 1984, *Nature* **310**, 22.

Eberhard, G., Schwarzschild, K.: 1913, *Astrophys. J.* **38**, 292.

Gilman, P.A., Howard, R.: 1984, *Astrophys. J.* **283**, 385.

Hill, F.: 1987, in *The Internal Solar Angular Velocity*, eds. B.R.Durney and S. Sofia, D. Reidel, Dordrecht.

Hoeksema, J.T.: 1984, Dissertation, CSSA-ASTRO-84-07, Stanford University, p. 188.

Howard, R., LaBonte, B.J.: 1980, *Astrophys. J.* **239**, L33.

Kleeorin, N.I., Ruzmaikin, A.A., Sokoloff, D.D.: 1983, *Astrophys. Space Sci.* **95**, 131.
Krause, F., Meinel, R.: 1987, submitted to *Geophys. Astrophys. Fluid Dyn.*
LaBonte, B.J., Howard, R.: 1982, *Solar Phys.* **75**, 161.
Leroy, J.-L., Noens, J.-C.: 1983, *Astron. Astrophys.* **120**, L1.
Makarov, V.I., Makarova, V.V.: 1986, *J. Astrophys. Astr.* **7**, 113.
Noyes, R.W., Weiss, N.O., Vaughan, A.H.: 1984, *Astrophys. J.* **287**, 769.
Rädler K.-H.: 1986a, in *Plasma-Astrophysics,* ESA SP-251, p.569.
Rädler K.-H.: 1986b, *Astron. Nachr.* **307**, 89.
Robinson, R.D., Durney, B.R.: 1982, *Astron. Astrophys.* **108**, 322.
Rüdiger, G.: 1980, *Astron. Nachr.* **301**, 181.
Rüdiger, G., Tuominen, I., Krause, F., Virtanen, H.: 1986, *Astron. Astrophys.* **166**, 306.
Rüdiger, G., Tuominen, I.: 1987, in *The Internal Solar Angular Velocity,* eds. B.R. Durney and S. Sofia, D. Reidel, Dordrecht, p. 361.
Schmitt, D.: 1987, *Astron. Astrophys.* **174**, 281.
Snodgrass, H.B., Howard, R.: 1985, *Solar Phys.* **95**, 221.
Steenbeck, M., Krause, F.: 1969, *Astron. Nachr.* **291**, 49.
Stenflo, J.O., Vogel, M.: 1986, *Nature* **319**, 285.
Stix, M.: 1974, *Astron. Astrophys.* **37**, 121.
Stix, M.: 1983, *Mitt. Astron. Ges.* **60**, 95.
Tuominen, I., Virtanen, H.: 1984, *Astron. Nachr.* **305**, 225.
Tuominen, I., Virtanen, H.: 1987, in *The Internal Solar Angular Velocity,* eds. B.R. Durney and S. Sofia, D. Reidel, Dordrecht, p.83.
Tuominen, J., Tuominen, I., Kyröläinen, J.: 1983, *Monthly Notices Roy. Astron. Soc.* **205**, 691.
Waldmeier, M.: 1935, *Astr. Mitt. Zürich* **14**, No.133, 105.
Wilson, O.C.: 1978, *Astrophys. J.* **226**, 379.
Yoshimura H.: 1975, *Astrophys. J. Suppl.* **29**, 467.
Yoshimura H.: 1976, *Solar Phys.* **50**, 3.