Rotational Effects on Reynolds Stresses in the Solar Convection Zone

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Abstract: Three-dimensional hydrodynamic simulations are carried out in a rectangular box. The angle between gravity and rotation axis is kept as an external parameter in order to study the latitude-dependence of convection. Special attention is given to the horizontal Reynolds stress and the Λ-effect (Rüdiger, 1989). The results of the simulations are compared with observations and theory and a good agreement is found.

1. Introduction

In traditional mean-field turbulence theory one writes the velocity as a sum of a mean-field part and a fluctuation around the mean, i.e. $\mathbf{u} = \langle \mathbf{u} \rangle + \mathbf{u}'$. This leads to the appearance of the Reynolds stress tensor $Q_{ij} = \langle u'_i u'_j \rangle$ in the equation of momentum. Q_{ij} may be expressed in terms of $\langle \mathbf{u} \rangle$. In the presence of rotation $\Omega = \langle u_\phi \rangle / r \sin\theta$ gives the dominant contribution. According to Λ-effect theory (Rüdiger 1989) the cross-correlations $Q_{r\phi}$ and $Q_{\theta\phi}$ can be written in the form

$$Q_{r\phi} = \nu_T \Omega \left(-\frac{r}{\Omega}\frac{\partial \Omega}{\partial r} + V^{(0)} + V^{(1)} \sin^2\theta + V^{(2)} \sin^4\theta + ...\right) \sin\theta, \tag{1}$$

$$Q_{\theta\phi} = \nu_T \Omega w_1 \left(1 + \frac{w_2}{w_1} \sin^2\theta + ...\right) \cos\theta \sin^2\theta. \tag{2}$$

In this work we compute the cross-correlations directly from numerical simulations and thus obtain the V- and w-coefficients. We solve the equations of mass, momentum, and energy by using a modified version of the code by Nordlund and Stein (1990):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \tag{3}$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \mathbf{g} - 2\Omega \times \mathbf{u} + \nabla \cdot \boldsymbol{\tau}, \tag{4}$$

$$\rho \frac{\partial e}{\partial t} + \rho(\mathbf{u} \cdot \nabla)e = \nabla \cdot (k\nabla T) - p\nabla \cdot \mathbf{u} + \Phi_{\mathrm{visc}}. \tag{5}$$

2. Method and results

We simulate stellar convection in a three-dimensional box which is located in the convective zone. The orientation of the box is such that its z-axis is parallel to gravity, the x-axis points opposite to the θ-direction, and the y-axis points towards east.

The box is divided into 31^3 mesh points at which the differential equations are stepped forward in time. In our model the density stratification is weak (density contrast is 1.5) and therefore, only part of the convective zone can be covered by the box. An important parameter is the Rossby number which here is close to unity. This is compatible with the value at the bottom of the solar convective zone.

The top and bottom boundaries are assumed to be impenetrable and stress free. We require a constant radiative flux at the bottom and assume the temperature to be fixed at the top. The boundaries in the x- and y-directions are periodic.

Table 1. Comparison of values of the w-coefficients

	w_1	w_2	w_2/w_1
Boussinesq-ansatz	-1.1	$+0.1$	-0.1
Virtanen (1989)	$+1.0$	$+5.6$	5.6
This work	$+0.2$	$+1.0$	5.0
Tuominen and Rüdiger (1989)	$+4.5$	$+2.9$	0.6

Seven different latitudes at the southern hemisphere are considered separately (-90, -75, -60, -45, -30, -15, and 0 degrees). $Q_{r\phi}$ ($\widehat{=} -Q_{yz}$) and $Q_{\theta\phi}$ ($\widehat{=} -Q_{xy}$) are calculated at these latitudes and the free parameters in equations (1) and (2) are adjusted to give the best fit. The error bars are a measure of the fluctuations. The fit for $Q_{r\phi}$ is shown in Fig. 1. In this fit the first term on the right-hand side of equation (1) was removed. At all depths of the box we find the same qualitative behaviour of the V-coefficients: $V^{(0)}$ and $V^{(2)}$ are negative and $V^{(1)}$ is positive. A similar result was obtained by Tuominen and Rüdiger (1989) at the middle of the convective zone. The fit for $Q_{\theta\phi}$ is shown in Fig. 2. In table 1 the ratio w_2/w_1 from our simulations is compared with those from other studies. The Boussinesq-ansatz does not include any Λ-effect while the work by Virtanen (1989, see also Tuominen, 1990) is a fit to Greenwich data and shows good agreement with the present work. Tuominen and Rüdiger base their numerical model on the differential rotation profile of Stenflo (1989) and also have positive w-coefficients, although their latitudinal profile of $Q_{\theta\phi}$ is somewhat different.

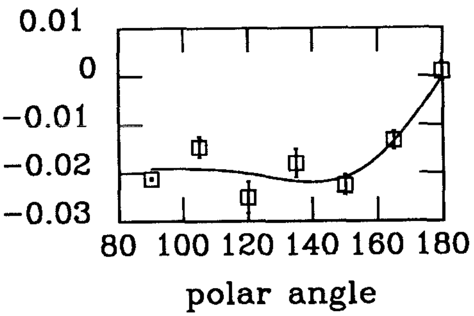

Fig. 1. The fit for $Q_{r\phi}$. The values from simulations (squares) were calculated at the mid-layer of the box.

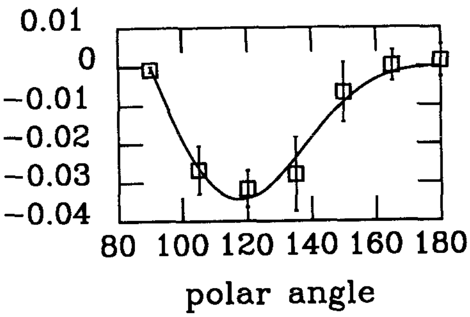

Fig. 2. The fit for $Q_{\theta\phi}$. The solid line is the fit and the squares denote the values from simulations which were calculated at the base of the box.

3. Conclusions

The main results of this work seem promising: the latitudinal dependence of the computed stresses show similar behaviour with the Λ-theory, and the horizontal Reynolds stress agrees with observations. The results for the horizontal Reynolds stress show that the Boussinesq-ansatz is insufficient as it gives the wrong sign for the horizontal cross-correlation. The Λ-theory introduces the V- and w- terms in equations (1) and (2) and gives the "correct" sign and shape for the latitude-dependence of the correlations. It should also be noticed that the production of this kind of effect does not need a global model. This suggests that the main generator of the Λ-effect is the Coriolis force. The presence of impenetrable boundary conditions in our model is, however, unrealistic and deserves further investigation in the future.

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